Chapter 3 MECHANICS

MECHANICS:

⇒ The branch where we study about motion is called mechanics.
⇒ There are two types of mechanics, kinematics and dynamics.
⇒ **Kinematics** is the sub branch of mechanics "where we study about the motion without cause(force).
⇒ **Dynamics** is another sub branch of mechanics "in which we study about motion with reference or that is caused or if their force involved.

A body

A certain amount of matter limited in all directions and consequently having a finite size, shape and occupying some definite space is called a body.

Particle

A particle is defined as a portion of matter infinitesimally small in size so that for the purpose of investigation, the distance between its different parts may be neglected. Thus, a particle has only a definite position, but no dimension. In the problems we are going to discuss, we will consider a body to be a particle for the sake of simplicity.

- In our daily life, we see lots of things moving around for example car passing through from one place to other, person riding on a bicycle and many more like this.
- In scientific terms an object is said to be in motion, if it changes its position with the description of time and if it does not change it position with the passage of time then it is said to be at rest.
- Both the motion and rest are relative terms.
  Let us explain above statement with the help of example where *mobile is kept on the table is resting at its position.*
  Now what do you think about this mobile: -
  (a)is it at rest or
  (b) is it moving i.e., changing its position with the passage of time.
  Now we have stated previously that:

Motion and rest are relative terms

☞ So for a person seeing mobile from earth it is at **rest**.
☞ Now consider a hypothetical scenario where we station our self on moon.
☞ Now from there earth seems to change its position with time (earth is rotating on its axis) and so mobile along with the table is changing its position.
☞ So, it is **moving** for a person tracking it from moon.

- Simplest case of motion is rectilinear motion which is the motion of the object in a straight line
- In our description of object, we will treat the object as a **point object**
- Object under consideration can be treated as point object if the size of the object is much smaller than the distance traveled by it in a reasonable time duration for example length of a motor car traveling a distance of 500 km can be neglected w.r.t distance traveled by it
- Here in **kinematics**, we study ways to describe the motion without going into the cause of the motion.

⇒ Rest and motion are relative not absolute, relative mean not fixed, and they depend upon the frame of reference.

EVENT

Anything that is happening which we are observing. E.g...

- Body is moving(event)
- Bomb is exploding(event)
- Chemical reaction(event)

⇒ Note that the average speed of earth is 30km/sec.

⇒ In universe nothing is absolute (exact).

Let say we are sitting, at chair we are at rest but with respect to moon we are in motion or simply we are in motion of 30km/sec.

⇒ Actually, reality is rarity/scarcity in world.

⇒ Science facing failure in determining the exact rest or motion.

Frame of Reference

⇒ A frame in which an observer is situated and makes his observations is known as his 'Frame of reference'.

⇒ The reference frame is associated with a co-ordinate system We can describe all the physical quantities like position, velocity, and acceleration etc... of an object in this coordinate system.

⇒ Frame of reference are of two types:

i. **inertial frame of reference**
ii. **Non-inertial frame of reference.**

i. Inertial frame of reference:

- A frame of reference which is at rest or which is moving with a uniform velocity along a straight line is called an inertial frame of reference.
- In inertial frame of reference Newton's laws of motion holds good.
- Inertial frame of reference is also called non-accelerated frame of reference or Newtonian or Galilean frame of reference.
- **Example**:

  The lift at rest, lift moving (up or down) with constant velocity, car moving with constant velocity on a straight road.

ii. Non-inertial frame of reference

- Accelerated frame of references are called non-inertial frame of reference.
- Newton's laws of motion are not applicable in non-inertial frame of reference.
- **Example:** Car moving in uniform circular motion, lift which is moving upward or downward with some acceleration

Position

⇒ To locate the position in motion or at rest, we need a frame of reference.

⇒ Simplest way to choose a frame of reference is to choose three mutually perpendicular axis labeled as X-, Y- and Z- axis as shown in figure below.



Figure 1: X, Y and Z axis

Distance and displacement:

- In the graph shown below an object is at position P at time t_1 and at position R at time t_2.
- In the time interval from t_1 to t_2 particle has traveled path PQR and length of the path PQR is the distance traveled by the object in the time interval t_1 to t_2
- Now connect the initial position of the object P with its final position R through a straight line and we get the displacement of the object.

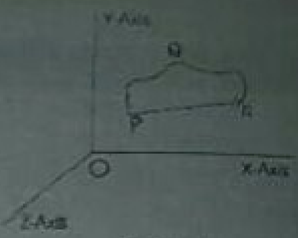


Figure 4

- Displacement of the object has both magnitude and direction i.e., displacement is a vector quantity.
- Magnitude of displacement vector is equal to the length of straight line joining initial and final position and its direction points from the initial position of object towards its final position.
- In contrast to displacement distance is scalar quantity.
- If a body is moving continuously in a given direction on a straight line, then the magnitude of displacement is equal to distance.
- Generally, the magnitude of displacement is less or equal to distance.
- Many paths are possible between two points. For different paths between two points, distances are different but magnitudes of displacement are same.
- The slope of distance-time graph is always greater or equal to zero.
- The slope of displacement-time graph may be negative.
- ***A straight-line distance Is called displacement.***
- For motion between two points, displacement is single valued while distance depends on actual path and so can have many values.
- For a moving particle distance can never decrease with time while displacement can. Decrease in displacement with time means body is moving towards the initial position.
- In general, magnitude of displacement is not equal to distance. However, it can be so if the motion is along a straight line without change in direction.
- If $|\vec{d}| \le d \rightarrow$ *for any path.*
- $|\vec{d}| = d \rightarrow$ *for single straigHt patH*
- $|\vec{d}| < d \rightarrow$ *for two different patH.*
- *the ratio of the displacement is;*

$$\frac{|\vec{d}|}{d} \le 1$$

Or;

$$\frac{d}{|\vec{d}|} \ge 1$$

For full circle	For semicircular path	For quarter circle
Distance = $2\pi r$ Displacement, d = 0 	$Distance, s = \pi r$ $Displacement, d = 2r$ 	$Distance, s = \frac{\pi r}{2}$ Displacement, d = Pythagoras theorem; $d^2 = r^2 + r^2 \rightarrow d = \sqrt{r^2 + r^2}$ $d = \sqrt{2}r^2 \rightarrow d = \sqrt{2r}$ Note that Pythagoras theorem; $\Rightarrow (hyp)^2 = (Base)^2 + (p_{erp})^2$  Semicircle

Some important MCQs For Concept

MCQ: A body covered a distance of 5 m along a semicircular path. The ratio of distance to displacement is?

Answer: The ratio between distance and displacement is $\frac{\pi}{2}$

Explanation: You know that your object covered a distance of 5 m along a semicircular path. The displacement of the object is the shortest distance between its starting point and its finish point.

The distance covered is equal to half of the length of a circle

$$distance = \frac{2\pi r}{2} = \pi r \ meters$$

Where r is the radius of a circle.

Let say the object started at the far left of the semi-circle and ended on the far right of the semi-circle. The shortest distance between these points is actually the diameter of the circle

$$displacement = 2r \ meters$$

You know that distance is equal to 5 m so you can write

$$distance - \pi r = 5 \Rightarrow = \frac{5}{\pi}$$

The ratio between distance and displacement will thus be

$$\frac{distance}{displacement} = \frac{5}{2r} = \frac{5}{2\frac{5}{\pi}} \rightarrow \frac{distance}{displacement} = \frac{5}{2\frac{5}{\pi}} = \frac{1}{\frac{2}{\pi}} = \frac{\pi}{2}$$

The ratio is unit less because you are dividing meter by meter which are cancel out.

MCQ: A body is moving along a circular path of radius r. What will be the distance and displacement of the body when it completes half a revolution?



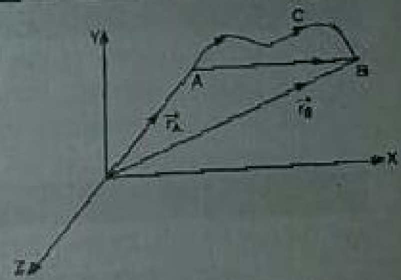
Answer:

1. Path '1' is the displacement as its shortest distance between start and end points. Hence **displacement** in this case is **2r**, where r is radius.

2. Path '2' is the actual path travelled which is nothing but distance.

Hence **distance** in this case is half of circumference i.e. πr

How to calculate:



Suppose an insect is at a point A (X_1, y_1, Z_1) at $t = t_1$ It reaches at point B (x_2, y_2, Z_2) at $t = t_2$ through path ACB with respect to the frame shown in figure. The actual length of curved path ACB is the distance travelled by the insect in time $\Delta t = t_2 - t_1$.

If we connect point A (initial position) and point B (final position) by a straight line, then the length of straight-line AB gives the magnitude of displacement of insect in time interval $\Delta t = t_2 - t_1$.

The direction of displacement is directed from A to B through the straight-line AB. From the concept of vector, the position vector of

$$\vec{A} = \vec{r_a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k} \text{ and that of } \vec{B} = \vec{r_b} = x_2\hat{i} + y_2\hat{j} + \vec{z_2}\hat{k}$$

According to addition law of vectors The magnitude of displacement,

$$\vec{r_a} = \vec{AB} = \vec{r_b}$$

$$\vec{AB} = \vec{r_a} - \vec{r_b}$$

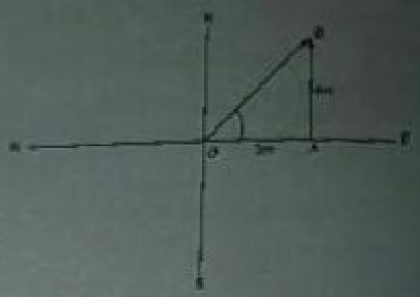
$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

The magnitude of displacement is

$$|\vec{AB}| = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2 + (Z_2 - Z_1)^2}$$

MCQ: A man walks 3 m in east direction, then 4 m north direction. Find distance covered and the displacement covered by man.

Solution:



The distance covered by man is the length of path = 3 m + 4 m = 7m.

Let the man starts from O and reaches finally at B (shown in figure). $\overrightarrow{OB}$ represents the displacement of man. From figure

$|\overrightarrow{OB}| = (OA)^2 + (AB)^2 \Rightarrow (3m)^2 + (4m)^2 = 5m$

And $\tan\theta = \frac{4m}{3m} = \frac{4}{3}$

$\theta = \tan^{-1}\left(\frac{4}{3}\right)$

The displacement is directed at an angle $\tan^{-1}\left(\frac{4}{3}\right)$ north of east.

MCQ: Which relation between distance and displacement magnitude is correct________?

a) $s > d$ b) $s \geq d$ (correct, in case of straight line) c) $s < d$ d) $s = 0, d > 1$

Explanation: $s \neq 0$, $d = 0 \rightarrow$ possible, $s = 0$, $d \neq 0 \rightarrow$ not possible

Comparison between distance and displacement

Distance	Displacement
For a moving particle in a given time interval distance can be many valued functions,	displacement would always be single valued function
Distance is always positive	Displacement may be positive, negative or zero.
distance can never decrease with time	Displacement can decrease with time
$\lvert\vec{d}\rvert \leq d \rightarrow$ *for any path.*	—
Distance would be equal to displacement if and only is particle is moving along straight line without any change in direction.	—

velocity

- Velocity is defined as the rate of change of displacement.
- It is a vector quantity, both magnitude and direction are required to define it.
- It's direction is same as that of displacement.
- SI unit of velocity is m/s

speed

- Speed is defined as the rate of change in distance with respect to time.
  1. It is a scalar quantity. Only magnitude is required to define the speed.
  2. Speed and velocity both have same unit.
- Velocity and speed are very important concepts in Kinematics.
- Now sometimes students ask questions like → *Is average speed = average velocity?*
- The answer is no, average speed is not equal to average velocity. This is because
  Average Speed = Total distance/Total time
  Average Velocity = Total displacement/Total time
- Just like both displacement and distance are different, average speed and average velocity are also different for ex. Motion around a circular track.
- Speedometer of a car measure instantaneous speed.

For a Circular path

for a close circular path,

$$\Rightarrow velocity = \vec{0} \rightarrow \vec{v} = 0 \text{ and magnitude of velocity: } |\vec{v}| = 0$$
$$\Rightarrow \text{but speed } \neq 0 \Rightarrow v \neq 0 \Rightarrow v = \frac{2\pi r}{T}$$

Why velocity is zero?

Because for a close circle the total displacement is zero, but distance is equal to circumference($2\pi r$) of a circle, now velocity is the time rate of change of displacement, so if displacement is zero the velocity will be also equal to zero, where speed is the time rate of change of distance, and distance is equal to circumference of a circle so speed will become;-> $v = \frac{2\pi r}{T}$

Average velocity

⇒ Consider a particle undergoing motion along a straight line i.e. particle is moving along X-axis.

⇒ Here in this case X co-ordinate describing motion of the particle from origin O varies with time or we can say that X co-ordinate depends on time.

⇒ If at time $t=t_1$ particle is at point P, at a distance x_1 from origin and at time $t=t_2$ it is at point Q at a distance x_2 from the origin then displacement during this time is a vector from point P to Q and is

$$\Delta x = x_2 - x_1 \quad (1)$$

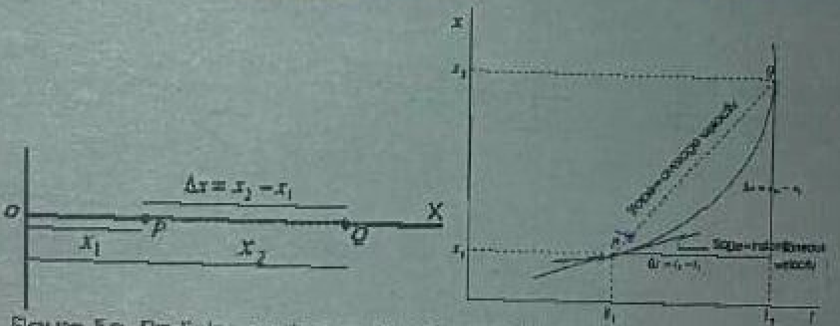


Figure 5a. Particle moving on X-axis Figure 5b. Co-ordinate time graph of the motion

- The average velocity of the particle is defined as the ratio of the displacement Δx of the particle in the time interval $\Delta t = t_2 - t_1$. If v_{avg} represents average velocity then,

$$v_{avg} = \frac{\Delta x}{\Delta t} \quad \rightarrow (2)$$

- Figure (5b) represents the co-ordinate time graph of the motion of the particle i.e., it shows how the value of x-coordinate of moving particle changes with the passage of time.
- In figure (5b) average velocity of the particle is represented by the slope of chord PQ which is equal to the ratio of the displacement Δx occurring in the particular time interval Δt.
- Like displacement average velocity v_{avg} also has magnitude as well as direction i.e., average velocity is a vector quantity.
- Average velocity of the particle can be *positive as well as negative* and its positive and negative value depends on the sign of displacement.
- If displacement of particle is zero its average velocity is also zero.
- Graphs below shows the x-t graphs of particle moving with positive, negative average velocity and the particle at rest.

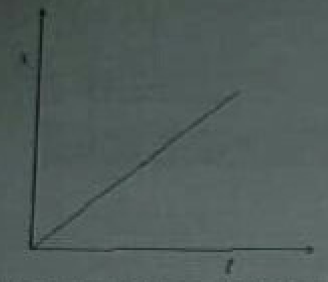


Figure 5c.Position time graph of particle moving with positive average velocity



Figure 5d.Position time graph of particle moving with negative average velocity

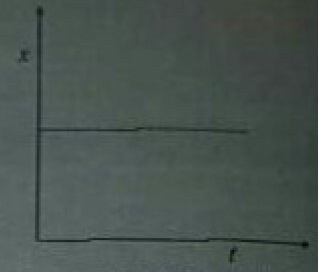


Figure 5e.Position time graph of particle moving with zero average velocity

average velocity definition

Average velocity is defined as the ratio of total displacement to total time. It is the average values of the velocities we know.

Average velocity formula:

This is extra section where we talk about **average velocity formula**. From average velocity definition we know that it is the ratio between total displacement and total time but this vav i.e., average velocity formula can have different form according to different problems. So before solving problems that asks you to find average velocity please check under which case your problem falls.

1. **Case 1:-**
   If X is total displacement of the body and t is the total time taken by the body to complete the displacement. The average velocity V_{av} is given by
   $$v_{avg} = \frac{Total\ distance\ travelled}{total\ time\ taken} = \frac{X}{T}$$
2. **Case 2:-**
   If we know that body have initial displacement position x_0 at time t_0 undergoes a motion and attains final displacement x_f at time t_f then, the average velocity is given as
   $$v_{av} = \frac{x_f - x_o}{t_f - t_o} = \frac{\Delta x}{\Delta t}$$
3. **Case 3:-**
   If we have a knowledge that $x_1, x_2, \ldots\ldots x_n$ are the distance covered by the body for time $t_1, t_2, \ldots\ldots t_n$. The average velocity is
   $$\frac{x_1 + x_2 \ldots\ldots\ldots\ldots\ldots + x_n}{t_1 + t_2 \ldots\ldots\ldots\ldots + t_n}$$
4. **Case 3:-**
   If a velocity is changing as a result of a constant acceleration, the average velocity can be found by adding the initial and final velocities, and dividing by 2. Here it is important to note that this formula applies only for constant acceleration case.
   Consider a body moving with uniform acceleration along a straight path. At some instant of time, initially its velocity is U m/s and at the end of t seconds let its final velocity be Vm/s. Hence the average velocity is given as
   $$V_{av} = \frac{v + u}{2}$$

How to derive this average velocity formula

Average velocity is displacement divided by time. Let us start with the formula for displacement and divide it by t.

$$s = ut + \frac{1}{2}at^2$$

$$average\ velocity = \frac{ut + \frac{1}{2}at^2}{t} \rightarrow u + \frac{1}{2}at \rightarrow \frac{2u + at}{2} \rightarrow \frac{u + (u + at)}{2}$$

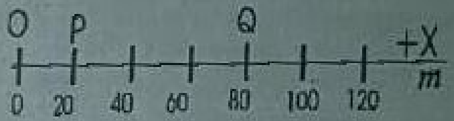
$$now.\ v = u + at\ Average\ velocity = \frac{u+v}{2}$$

Average Speed

- From graph 5c it is clear that for positive average velocity slope of line slants upwards right or we can say that it has positive slope.
- For negative average velocity slope line slants upwards down to the right i.e. it has negative slope.
- For particles at rest slope is zero.
- So far we have learned that Average speed is defined as total distance traveled divided by time taken.
- Displacement of the object is different from the actual distance traveled by the particle.
- For actual distance traveled by the particle its average speed is defined as the total distance traveled by the particle in the time interval during which the motion takes place.
- Mathematically,

$$Average\ speed = \frac{Total\ distance\ travelled}{Total\ time\ interval} \rightarrow (3)$$

- Since distance traveled by an particle does not involve direction so speed of the particle depending on distance traveled does not involve direction and hence is a scalar quantity and is always positive.
- Magnitude of average speed may differ from average velocity because motion in case of average speed involve distance which may be greater than magnitude of displacement



Let us consider an example

Figure 6

here a man starts traveling from origin till point Q and return to point P then in this case displacement of man is

Displacement from O to Q is OQ=80m

Displacement from Q to P is =20m-80m=-60m

total displacement of particle in moving from O to Q and then moving Q to P is = 80m + (-60m) = 20 m

Now total distance traveled by man is OQ+OP=80m+60m=140m

Hence during same course of motion distance traveled is greater then displacement.

- From this we can say that average speed depending on distance is in general greater than magnitude of velocity.

Average speed in different situation

1. An object travels different distances with different speeds is same direction;

$$Total\ distance = S_1 + S_2 + S_3$$

$$Total\ time = \frac{S_1}{v_1} + \frac{S_2}{v_2} + \frac{S_3}{v_3}$$

$$Average\ speed = \frac{S_1}{\frac{S_1}{v_1}} + \frac{S_2}{\frac{S_2}{v_2}} + \frac{S_3}{\frac{S_3}{v_3}}$$

2. If object travels two equal distances $(S_1 + S_2 = S)$ with different speed $v_1 and\ v_2$

$$Average\ speed = \frac{S+S}{\frac{S}{v_1} + \frac{S}{v_2}} = \frac{2v_1v_2}{v_1 + v_2}$$

3. An object travels with different speed during different times intervals,

$$Total\ distance = v_1t_1 + v_2t_2 + v_3t_3$$
$$Total\ time\ taken = t_1 + t_2 + t_3$$
$$Average\ speed = \frac{v_1t_1 + v_2t_2 + v_3t_3}{t_1 + t_2 + t_3}$$

4. If $t_1 = t_2 = t_3 = t$;

$$Average\ speed = \frac{v_1 + v_2 + v_3}{3}$$

We use average speed when an object moves with a variable speed, we describe its motion in terms of average speed.

Instantaneous velocity and speed

- Velocity of particle at any instant of time or at any point of its path is called instantaneous velocity.
- Again, consider the graph 5b and imagine second point Q being taken more and more closer to point P then calculate the average velocity over such short displacement and time interval.
- Instantaneous velocity can be defined as limiting value of average velocity when second point comes closer and closer to the first point.
- Limiting value of $\Delta x / \Delta t$ as Δt approaches zero is written as dx/dt, and is known as instantaneous velocity. Thus, instantaneous velocity is

$$v = \lim_{\Delta t \to \infty} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- As point Q approaches point 'P' in figure 5a in this limit slope of the chord PQ becomes equal to the slope of tangent to the curve at point P.
- Thus, we can say that instantaneous velocity at any point of a coordinate time graph is equal to the slope of the tangent to the graph at that point.
- Instantaneous speed or speed is the magnitude of the instantaneous velocity unlike the case of average velocity and average speed where average speed over a finite interval of time may be greater than or equal to average velocity.
- Unit of average velocity, average speed, instantaneous velocity and instantaneous speed is ms^{-1} in SI system of units.
- Some other units of velocity are $ft\,s^{-1}$, $cm\,s^{-1}$.

MCQ: if a car moves with the speed 40 m/s for 1st 30 minutes and then moves with speed 60 m/sec for next 30 min then average speed________??

a) 40 b) 50 c) 48 d) 60

EXPLANATION:

$$\Rightarrow \frac{v_1 + v_2}{2} \Rightarrow \frac{40 + 60}{2} \Rightarrow \frac{100}{2} = 50m/s$$

MCQ: If a car moves from Arsalan's home to Islamabad with speed of 40m/s and then return back from Islamabad to Arsalan's home through same path with the speed 60 m/s their average speed_______?

EXPLANATION:

$$\Rightarrow \frac{2v_1v_2}{v_1 + v_2} \Rightarrow \frac{2(40)(60)}{(40) + (60)} \Rightarrow 48\ m/s$$

MCQ: if Arsalan is driving an aero plane and it moves with speed 5m/s for 3 seconds and then further moves with speed 10 m/sec for 6 sec its average speed will be__________?

a) 5 m/sec b) 10 m/sec c) 8.3 m/sec d) 20.1 m/sec

EXPLANATION:

$$\Rightarrow \frac{Total\ distance}{Total\ time} \Rightarrow \frac{75}{9} \Rightarrow 8.333\ m/s$$

Acceleration

⇒ Acceleration is the rate of change of velocity with time.

⇒ For describing average acceleration, we first consider the motion of an object along X-axis.

⇒ Suppose at time t_1 object is at point P moving with velocity v_1 and at time t_2 it is at point Q and has velocity v_2. Now average acceleration of object in moving from P to Q is

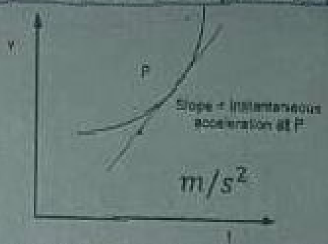
$$a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

which is the change in velocity of object with the passage of time.

⇒ Instantaneous acceleration can be defined in the same way as instantaneous velocity

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

⇒ The instantaneous acceleration at any instant is the slope of v-t graph at that instant.



⇒ In figure; instantaneous acceleration at point P is equal to the slope of tangent at this point P.

⇒ Since velocity of a moving object has both magnitude and direction likewise acceleration depending on velocity has both magnitude and direction and hence acceleration is a vector quantity.

⇒ For circular path acceleration is perpendicular to the velocity, acceleration depends upon change in velocity.

⇒ Acceleration may be positive, negative or zero.

Positive acceleration:	Negative acceleration:
when velocity increased then acceleration and velocity are parallel and acceleration is perpendicular to the force.	when velocity decrease acceleration and velocity are anti parallel and acceleration is parallel to the force. F= ma

⇒ Acceleration is always in the direction of force (only parallel)

⇒ SI unit of acceleration is ms^{-2}

Some important points

⇒ We know that,

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

- $if\ v_f > v_i\ (speed\ up) then\ \Delta\vec{v} = positive, so\ \vec{a} = +ve, acceleration\ will\ be\ in\ direction\ of\ motion.$
- $if\ v_f < v_i\ (slow\ down) then\ \Delta\vec{v} = -ve, and\ so\ \vec{a} = -ve, acceleration\ will\ be\ opposite\ to\ motion.$

⇒ **MCQ POINT:** $|\vec{v}_{ins}| = speed \rightarrow for\ circular\ motion.$

- The slowdown of a body is also called "Retardation" or "Deceleration". But there is some difference between these two words.
- Retardation is the term used when "in small time large decrease occur (non-uniform decrease)".
- Deceleration is the term used when "there is uniform (equal) decrease.

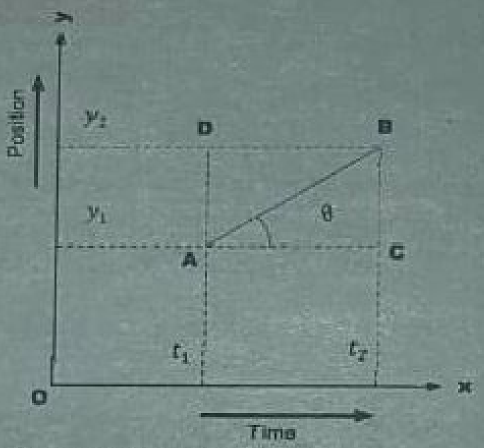
Possible ways of velocity change:

When only direction of velocity changes	When only magnitude of velocity changes	When both magnitude and direction of velocity changes
Acceleration perpendicular to velocity	Acceleration parallel or anti-parallel to velocity	Acceleration has two components one is perpendicular to velocity and another parallel or anti-parallel to velocity
Ex... Uniform circular motion	Ex... Motion gravity under	Ex... Projectile motion

Position time Graph:

During motion of the particle its parameters of kinematical analysis (v, a, s) changes with time. This can be represented on the graph

Position time graph is plotted by taking time t along x-axis and position of the particle on y-axis.



Let AB is a position-time graph for any moving particle

$$As\ velocity = \frac{change\ in\ posiition}{time\ taken} = \frac{y_2 - y_1}{t_2 - t_1} \quad ...(i)$$

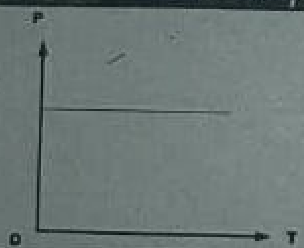
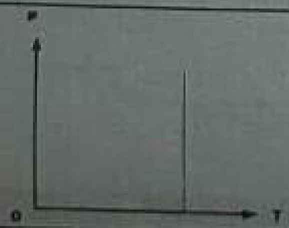
$$For\ a\ triangle\ ABC;\ tan\theta = \frac{BC}{AC} = \frac{AD}{AC} = \frac{y_2 - y_1}{t_2 - t_1} \quad ...(ii)$$

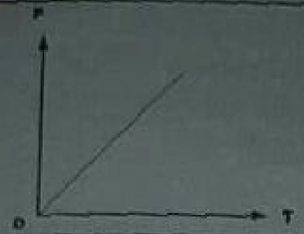
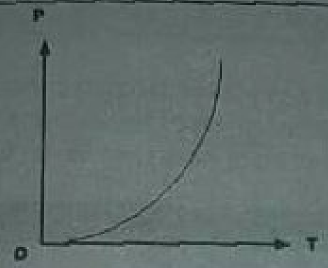
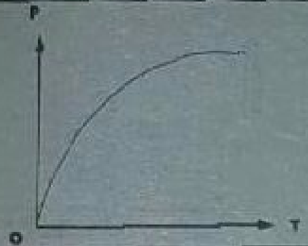
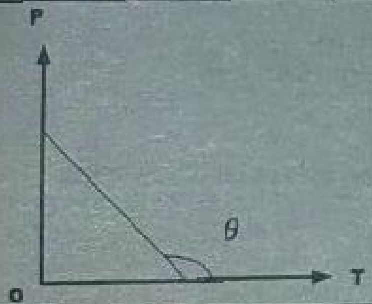
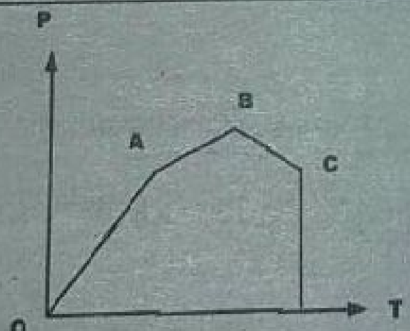
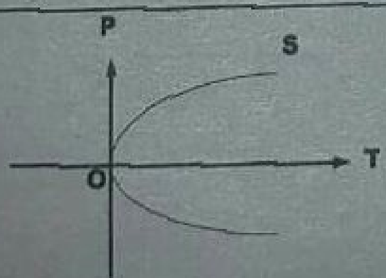
By comparing (i) and (ii) Velocity = tanθ

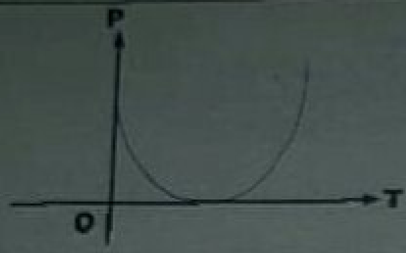
$v = \tan\theta$

It is clear that slope of tangent on position-time graph represents the velocity of the particle.

Various position -time graphs and their interpretation:

	θ = 0° so v = 0 i.e., line parallel to time axis represents that the particle is at rest.
	θ = 90° so v = ∞ i.e., line perpendicular to time axis represents that particle is changing its position but time does not change it means the particle possesses infinite velocity. Practically this is not possible.

	θ = constant so v = constant, a = 0 i.e., line with constant slope represents uniform velocity of the particle.
	θ is increasing so v is increasing, a is positive. i.e., line bending towards position axis represents increasing velocity of particle. It means the particle possesses acceleration.
	θ is decreasing so v is decreasing, a is negative i.e., line bending towards time axis represents decreasing velocity of the particle. It means the particle possesses retardation.
	θ constant but > 90° so v will be constant but negative i.e., line with negative slope represent that particle returns towards the point of reference. (negative displacement)
	Straight line segments of different slopes represent that velocity of the body changes after certain interval of time.
	This graph shows that at one instant the particle has two positions, which is not possible

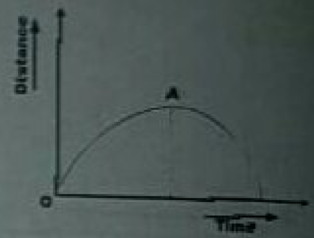
	The graph shows that particle coming towards origin initially and after that it is moving away from origin.

NOTE: If the graph is plotted between distance and time then it is always an increasing curve and it never comes back towards origin because distance never decreases with time. Hence such type of distance time graph is valid up to point A only, after point A, it is not valid as shown in the figure

Velocity-time Graph:

The graph is plotted by taking time t along x-axis and velocity of the particle on y-axis.

Calculation of Distance and displacement: The area covered between the velocity time graph and time axis gives the displacement and distance travelled by the body for a given time interval.



$$total\ distance = |A_1| + |A_2| + |A_3|$$

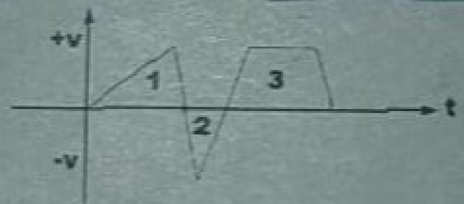
$$= Addition\ of\ modulus\ of\ different\ area, i.e \ldots s = \int |v| dt$$

$$total\ displacement = A_1 + A_2 + A_3$$

$$= Addition\ of\ different\ area\ considering\ their\ sign$$

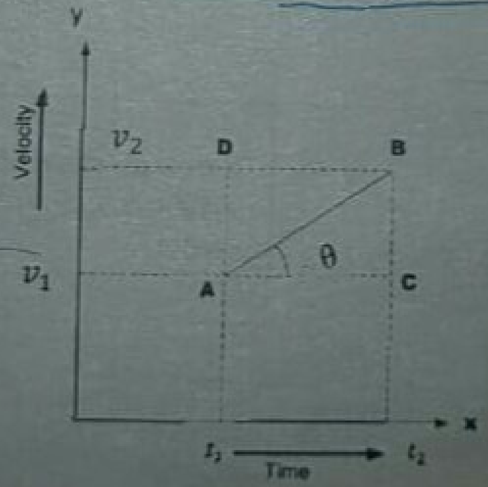
$$i.e..r = \int vdt$$

Area above time axis is taken as positive, while area below time axis is taken as negative



Here A_1 and A_2 are area of triangle 1 and 2 respectively and A_3 is the area of trapezium.

Calculation of Acceleration: Let AB is a velocity-time graph for any moving particle

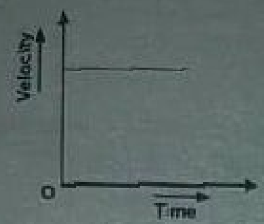
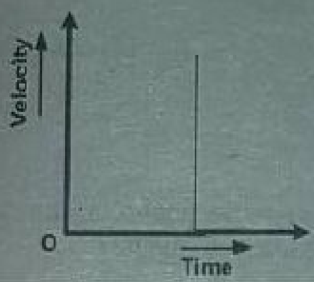
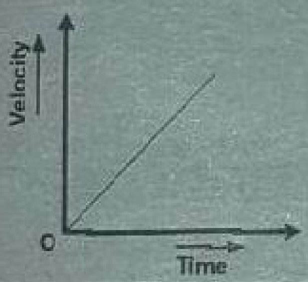
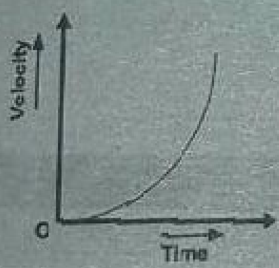


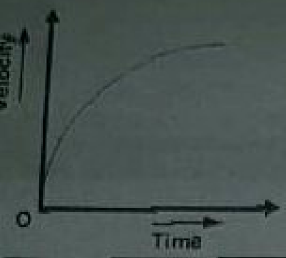
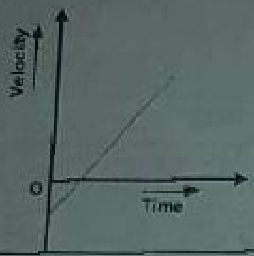
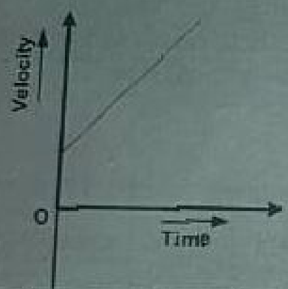
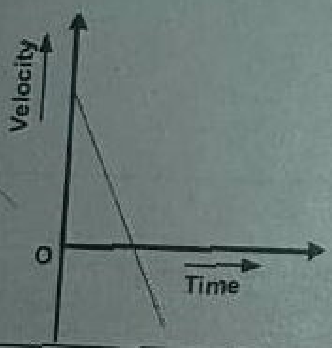
$$As\ acceleration = \frac{change\ in\ velocity}{time\ taken} = \frac{v_2 - v_1}{t_2 - t_1} \quad (I)$$

$$for\ triangle\ ABC, \tan\theta = \frac{BC}{AC} = \frac{AD}{AC} = \frac{v_2 - v_1}{t_2 - t_1} \quad (II)$$

Acceleration (a) = $\tan\theta$

It is clear that slope of tangent on velocity-time graph represents the acceleration of the particle.

Various velocity-time graphs and their interpretation:	
	$\theta = 0°$, a = 0, v = constant i.e., line parallel to time axis represents that the particle is moving with constant velocity
	$\theta = 90°$, $a = \infty$, v = increasing i.e., line perpendicular to time axis represents that the particle is increasing its velocity, but time does not change. It means the particle possesses infinite acceleration. Practically it is not possible
	θ = constant, so a = constant and v is increasing uniformly with time i.e., line with constant slope represents uniform acceleration of the particle.
	θ increasing so acceleration increasing i.e., line bending towards velocity axis represent the increasing acceleration in the body.

	θ decreasing so acceleration decreasing i.e. line bending towards time axis represents the decreasing acceleration in the body
	Positive constant acceleration because θ is constant and < 90° but initial velocity of the particle is negative.
	Positive constant acceleration because θ is constant and < 90° but initial velocity of particle is positive.
	Negative constant acceleration because θ is constant and > 90° but initial velocity of the particle is positive.

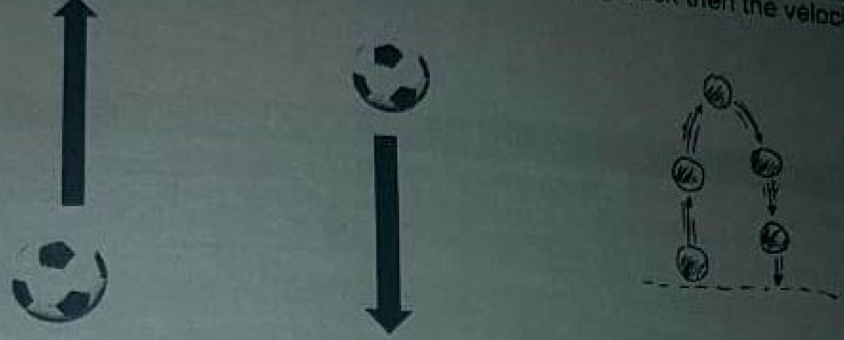
SUMMARY:

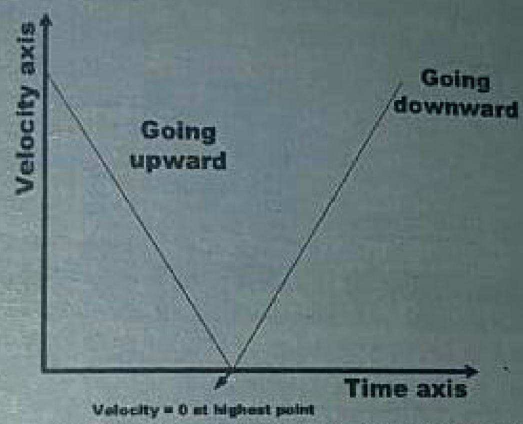
- ⇒ The displacement remains unaffected due to shifting of origin from one point to the other.
- ⇒ The displacement can have positive, negative or zero value.
- ⇒ The displacement is never greater than the actual distance travelled.
- ⇒ The displacement has unit of length.
- ⇒ Velocity can be considered to be a combination of speed and direction.
- ⇒ A change in either speed or direction of motion results in a change in velocity.
- ⇒ It is not possible for a particle to possess zero speed with a non-zero velocity.
- ⇒ A particle which completes one revolution, along a circular path, with uniform speed is said to possess zero velocity and non-zero speed.

⇒ In case a body moves with uniform velocity, along a straight line, its average speed is equal to its instantaneous speed.
⇒ During free fall motion the acceleration is constant.

SOME Other IMPORTANT GRAPHS:

⇒ Let suppose we have a ball if the ball is thrown upward and falling back then the velocity time graph for this ball will be;

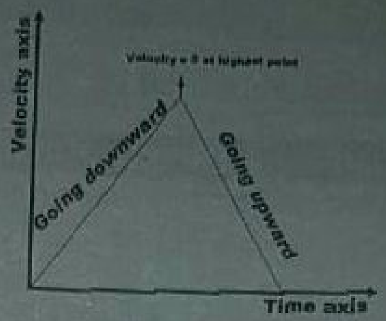




- The velocity will be maximum when the ball is initially at ground.
- The velocity goes on decreasing as the ball moves upward and become zero at heighest point.
- Again velocity goes on increasing when the ball moves downward back to earth and at the earth the velocity of the ball is again maximum.

⇒ Let suppose we have a ball if the ball is thrown / hatted by the ground and bouncing back then the velocity time graph for this ball will be;





DEFINATION OF THE SLOPE:

⇒ The slope of a line in the plane containing the *x* and *y* axes is generally represented by the letter *m*, and is defined as the change in the *y* coordinate divided by the corresponding change in the *x* coordinate, between two distinct points on the line. This is described by the following equation:

$$m = \frac{\Delta y}{\Delta x} = \frac{vertical\ change}{horizental\ channge} = \frac{rise}{run}$$

(The Greek letter *delta*, Δ, is commonly used in mathematics to mean "difference" or "change".) Given two points (x_1,y_1) and (x_2,y_2), the change in *x* from one to the other is $x_2 - x_1$ (*run*), while the change in *y* is $y_2 - y_1$ (*rise*). Substituting both quantities into the above equation generates the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

⇒ The formula fails for a vertical line, parallel to the *y* axis (Division by zero), where the slope can be taken as infinite, so the slope of a vertical line is considered undefined.

Summary: Some Important Points:

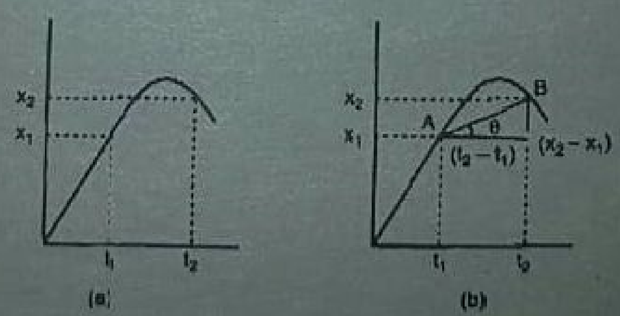
⇒ Velocity is a vector quantity while speed is a scalar quantity.

⇒ If a particle travels equal distances at speeds v1, v2, v3, ... etc. respectively, then the average speed is harmonic mean of individual speeds.

⇒ If a particle moves a distance at speed V1 and comes back with speed v2, then $v_{av} = \frac{2v_1v_2}{v_1+v_2}$. **But** $v_{av} = 0$.

⇒ If a particle moves in two equal intervals of time at different speeds v1 and v2 respectively, then $v_{av} = \frac{v_1v_2}{2}$.

⇒ The average velocity between two points in a time interval can be obtained from a position versus
time graph by calculating the slope of the straight line joining the co-ordinates of the two points.



The graph [shown in fig. (a)], describes the motion of a particle moving along x-axis (along a ...ght
...uppose we wish to calculate the average velocity between t = tj and t = t2. The slope of

chord AB
[shown in fig. (b)] gives the average velocity.

Mathematically: $v_{av} = \tan\theta = \frac{x_2 - x_1}{t_2 - t_1}$

- ✓ The area of speed-time graph gives distance.
- ✓ The area of velocity-time graph gives displacement.
- ✓ Speed can never be negative.
- ✓ Average velocity may or may not be equal to instantaneous velocity.
- ✓ If body moves with constant velocity, the instantaneous velocity is equal to average velocity.
- ✓ The instantaneous speed is equal to modulus of instantaneous velocity.
- ✓ Distance travelled by particle is $s = \int |V|\, dt$.
- ✓ If particle moves on a straight line, (along x-axis), then $v = \frac{dx}{dt}$
- ✓ The area of velocity-time graph gives displacement.
- ✓ The area of speed-time graph gives distance.
- ✓ The slope of tangent at position-time graph at a particular instant gives instantaneous velocity at that instant.
- ✓ Acceleration a vector quantity.
- ✓ Its unit is m/s.
- ✓ The slope of velocity-time graph gives acceleration.
- ✓ The area of acceleration-time graph in a particular time interval gives change in velocity in that time interval.

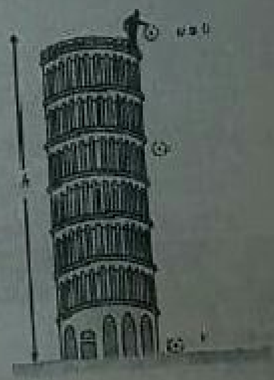
Some other IMPORTANT POINTS:

- Acceleration is always directed downward.
- All bodies, when dropped (u=0) from same height should fall with same rapidity and should take same time to reach the earth.

Equation of Kinematics:

These are the various relations between u, v, a, t and s for the particle moving with uniform acceleration where the notations are used as:

u = Initial velocity of the particle at time $t = 0$ sec
v = Final velocity at time t sec
a = Acceleration of the particle
s = Distance travelled in time t sec
s_n = Distance travelled by the body in nth sec



(1) When particle moves with zero acceleration

(i) It is a unidirectional motion with constant speed.
(ii) Magnitude of displacement is always equal to the distance travelled.
(iii) $v = u$, $s = ut$ [As $a = 0$]

(2) When particle moves with constant acceleration

(i) Acceleration is said to be constant when both the magnitude and direction of acceleration remain constant.
(ii) There will be one dimensional motion if initial velocity and acceleration are parallel or anti-parallel to each other.

(iii) Equations of motion (in scalar from)	Equation of motion (in vector from)
$v = u + at$	$\vec{v} = \vec{u} + \vec{a}t$
$s = ut + \frac{1}{2}at^2$	$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$
$v^2 = u^2 + 2as$	$\vec{v}.\vec{v} = \vec{u}.\vec{u} + 2\vec{a}.\vec{s}$

$s = (\frac{u+v}{2})t$	$\vec{s} = \frac{1}{2}(\vec{u} + \vec{v})t$
$s_n = u + \frac{a}{2}(2n-1)$	$\vec{s_n} = \vec{u} + \frac{\vec{a}}{2}(2n-1)$

Motion of Body under Gravity (Free Fall):

The force of attraction of earth on bodies, is called force of gravity. Acceleration produced in the body by the force of gravity, is called acceleration due to gravity. It is represented by the symbol g. In the absence of air resistance, it is found that all bodies (irrespective of the size, weight or composition) fall with the same acceleration near the surface of the earth. This motion of a body falling towards the earth from a small altitude (h << R) is called free fall. An ideal example of one-dimensional motion is motion under gravity in which air resistance and the small changes in acceleration with height are neglected.

(1) If a body is dropped from some height (initial velocity zero)

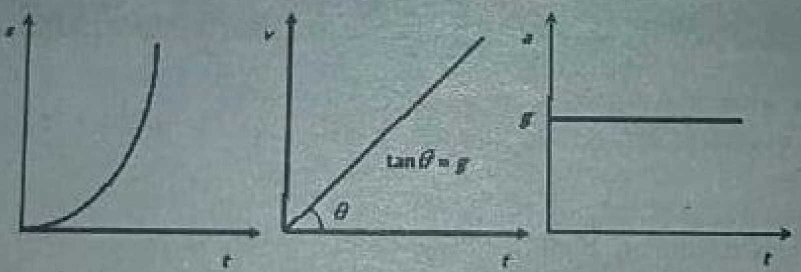
⇒ ***Equations of motion:*** Taking initial position as origin and direction of motion (i.e., downward direction) as a positive, here we have

$$t = \sqrt{\frac{2h}{g}} = \frac{v}{g} \Rightarrow v = \sqrt{2hg} \Rightarrow h = \frac{v^2}{2g}$$

$$u = 0 \text{ and } a = +g,$$

$$v = gt \rightarrow (i),\ h = \frac{1}{2}gt^2 \rightarrow (ii),\quad v^2 = 2gh \rightarrow (iii) \text{and } h_n = \frac{g}{2}(2n-1) \rightarrow (iv)$$

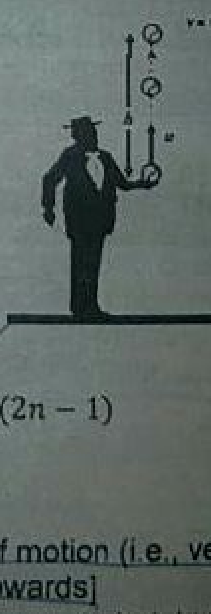
⇒ ***Graph of distance, velocity and acceleration with respect to time***



⇒ As h = (1/2)gt², i.e., h ∝ t², distance covered in time t, 2t, 3t, etc., will be in the ratio of 12 : 22 : 32, i.e., square of integers.

⇒ *(iv)* The distance covered in the nth sec, $h_n = \frac{1}{2}g(2n-1)$

So distance covered in 1st, 2nd, 3rd sec, etc., will be in the ratio of 1 : 3 : 5, i.e., odd integers only



$$t = \sqrt{\frac{2h}{g}} = \frac{u}{g}$$

$$u = \sqrt{2gh}$$

$$h = \frac{u^2}{2g}$$

(2) If a body is projected vertically downward with some initial velocity

Equation of motion : $v = u + gt$

$$h = ut + \frac{1}{2}gt^2 \quad v^2 = u^2 + 2gh \text{ so,} \qquad h_n = u + \frac{g}{2}(2n-1)$$

(3) If a body is projected vertically upward

⇒ ***Equation of motion:*** Taking initial position as origin and direction of motion (i.e., vertically up) as positive a = − g [As acceleration is downwards while motion upwards]

So, if the body is projected with velocity u and after time t it reaches up to height h then

$$v = u - gt; \quad h = ut - \frac{1}{2}gt^2; \quad v^2 = u^2 - 2gh; \quad h_n = u - \frac{g}{2}(2n - 1)$$

⇒ For maximum height v = 0 So from above equation $u = gt$.

$$h = \frac{1}{2}gt^2 \text{ and } u^2 = 2gh$$

It is clear that both quantities do not depend upon the mass of the body or we can say that in absence of air resistance, all bodies fall on the surface of the earth with the same rate.

(4) The motion is independent of the mass of the body, as in any equation of motion, mass is not involved. That is why a heavy and light body when released from the same height, reach the ground simultaneously and with same velocity i.e., $t = \sqrt{2h/g}$ and $v = \sqrt{2gh}$

(5) In case of motion under gravity, time taken to go up is equal to the time taken to fall down through the same distance. Time of descent (t^2) = time of ascent ($t1$) = u/g

$$\textit{Total time of flight},\ T = t_1 + t_2 = \frac{2u}{g}$$

(6) In case of motion under gravity, the speed with which a body is projected up is equal to the speed with which it comes back to the point of projection. As well as the magnitude of velocity at any point on the path is same whether the body is moving in upwards or downward direction.

(7) A body is thrown vertically upwards. If air resistance is to be taken into account, then the time of ascent is less than the time of descent. $t_2 > t_1$

Let u is the initial velocity of body then time of ascent $t_1 = \frac{u}{g+a}$ and $h = \frac{u^2}{2(g+a)}$

where g is acceleration due to gravity and a is retardation by air resistance and for upward motion both will work vertically downward. For downward motion a and g will work in opposite direction because a always work in direction opposite to motion and g always work vertically downward

$$so\ h = \frac{1}{2}(g - a)t_2^2 \Rightarrow \frac{u^2}{2(g + a)} = \frac{1}{2}(g - a)t_2^2 \Rightarrow t_2 = \frac{u}{\sqrt{(g+a)(g-a)}}$$

Comparing t_1 and t_2 we can say that $t_2 > t_1$ since $(g + a) > (g - a)$

Galileo and Newton prediction



A hovercraft, such as the one in Figure, glides along the surface of the water on a cushion of air. A common misconception is that an object on which no force is acting will always be at rest. This situation is not always the case. If the hovercraft shown in Figure is moving at a constant velocity, then there is no net force acting on it. To see why this is the case, consider how a block will slide on different surfaces. First, imagine a block on a deep, thick carpet. If you apply a force by pushing the block, the block will begin sliding, but soon after you remove the force, the block will come to rest. Next, imagine pushing the same block across a smooth, waxed floor. When you push with the same force, the block will slide much farther before coming to rest. In fact, a block sliding on a perfectly smooth surface would slide forever in the absence of an applied force. In the 1630s, Galileo concluded correctly that it is an object's nature to maintain its state of motion or rest. Note that an object on which no force is acting is not necessarily at rest; the object could also be moving with a constant velocity. This concept was further developed by Newton in 1687 and has come to be known as Newton's first law of motion. is the tendency of an object not to accelerate. Newton's first law is often referred to as the law of inertia because it states that in the absence of a net force, a body will preserve its state of motion. In other words, Newton's first law says that when the net external force on an object is zero, the object's acceleration (or the change in the object's velocity) is zero.

Point mass:

⇒ An object can be considered as a point object if during motion in a given time, it covers distance much greater than its own size.

⇒ Object with zero dimension considered as a point mass.

⇒ Point mass is a mathematical concept to simplify the problems.

Newton's Laws

NEWTON'S FIRST LAW

'An object at rest remains at rest, and an object in motion continues in motion with constant velocity (that is, constant speed in a straight line) unless the object experiences a net external force'

If Net force on object is zero then:

- object at rest stay at rest.
- object in motion continue to move.

Inertia:

⇒ It is the tendency of an object not to accelerate. Newton's first law is often referred to as the *law of inertia* because

⇒ it states that in the absence of a net force, a body will preserve its state of motion.

⇒ In other words, Newton's first law says that *when the net external force on an object is zero the object's acceleration (or the change in the object's velocity) is zero.*

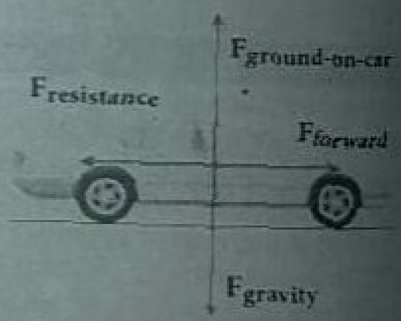
⇒ Inertia is not a physical quantity, it is only a property of the body which depends on mass of the body.

⇒ Inertia has no units and no dimensions.

⇒ Two bodies of equal mass, one in motion and another is at rest possess same inertia because it is a factor of mass only and does not depend upon the velocity.

The sum of forces acting on an object is the net force

Consider a car traveling at a constant velocity. Newton's first law tells us that the net external force on the car must be equal to zero. However, **Figure** shows that many forces act on a car in motion. The vector $F_{forward}$ represents the forward force of the road on the tires. The vector $F_{resistance}$, which acts in the opposite direction, is due partly to friction between the road surface and tires and is due partly to air resistance. The vector $F_{gravity}$ represents the downward gravitational force on the car, and the vector $F_{ground-on-car}$ represents the upward force that the road exerts on the car. To understand how a car under the influence of so many forces can maintain a constant velocity, you must understand the distinction between external force and net external force.



An *external force* is a single force that acts on an object as a result of the interaction between the object and its environment. All four forces in **Figure** are external forces acting on the car. The is the vector sum of all forces acting on an object. When all external forces acting on an object are known, the net force can be found by using the methods for finding resultant vectors. The net force is equivalent to the one force that would produce the same effect on the object that all of the external forces combined would. Although four forces are acting on the car in **Figure,** the car will maintain its constant velocity as long as the vector sum of these forces is equal to 0.

Although several forces are acting on this car, the vector sum of the forces is zero, so the car moves at a constant velocity.

Net force

a single force whose external effects on a rigid body are the same as the effects of several actual forces acting on the body.

Mass is a measure of inertia: Imagine a basketball and a bowling ball at rest side by side on the ground. Newton's first law states that both balls remain at rest as long as no net external force acts on them. Now, imagine supplying a net force by pushing each ball. If the two are pushed with equal force, the basketball will accelerate much more than the bowling ball. The bowling ball experiences a smaller acceleration because it has more inertia than the basketball does. As the example of the bowling ball and the basketball shows, the inertia of an object is proportional to the object's mass. The greater the mass of a body, the less the body accelerates under an applied force. Similarly, a light object undergoes a larger acceleration than does a heavy object under the same force. Therefore, mass, which is a measure of the amount of matter in an object, is also a measure of the inertia of an object.

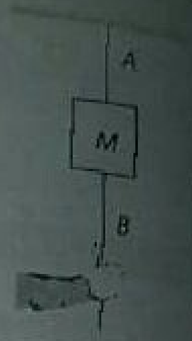
Inertia and Mass

- From First law of motion an object at rest would not move unless it is acted upon by a force
- This inherent property of objects to remain at rest unless acted upon by a force is called inertia rest
- Now consider the case of an object moving with uniform velocity along the straight line Again from Newton's law it would continue to move with uniform
- This inherent property by virtue of which a body in state of uniform motion tend to maintain its uniform motion is called inertia of motion
- Combining these two statements 'The property of an object to remain in state of rest or uniform rectilinear motion unless acted upon by a force is called inertia'
- Mass of any body is the measure of inertia. For example if we apply equal amount of force on two objects of different mass (say m_1 and m_2 such that $m_1 > m_2$) then acceleration of both the object would be different (i.e $a_1 < a_2$)
- Acceleration of object having larger mass would be lesser then the acceleration of object having smaller mass
- Thus, larger the mass of the body, smaller would be the acceleration and larger would be the inertia
- Newton's first law of motion revealing this fundamental property of matter i.e inertia is also known as law of inertia

⇒ Newton's first law defines inertia and is rightly called the law of inertia. Inertia are of three types:

1. Inertia of rest,
2. Inertia of motion,
3. Inertia of direction

1. INERTIA OF REST: It is the inability of a body to change by itself, its state of rest. This means a body at rest remains at rest and cannot start moving by its own **EXAMPLES;**
i. A person who is standing freely in bus, thrown backward, when bus starts suddenly. When a bus suddenly starts, the force responsible for bringing bus in motion is also transmitted to lower part of body, so this part of the body comes in motion along with the bus. While the upper half of body (say above the waist) receives no force to overcome inertia of rest and so it stays in its original position. Thus there is a relative displacement between the two parts of the body and it appears as if the upper part of the body has been thrown backward
ii. When a horse starts suddenly, the rider tends to fall backward on account of inertia of rest of upper part of the body as explained above.
iii. A bullet fired on a window pane makes a clean hole through it while a stone breaks the whole window because the bullet has a speed much greater than the stone. So it's time of contact with glass is small. So in case of bullet the motion is transmitted only to a small portion of the glass in that small time. Hence a clear hole is created in the glass window, while in case of ball, the

time and the area of contact is large. During this time the motion is transmitted to the entire window, thus creating the cracks in the entire window.
iv. In the arrangement shown in the figure. (a) If the string B is pulled with a sudden jerk then it will experience tension while due to inertia of rest of mass M this force will not be transmitted to the string A and so the string B will break. (b) If the string B is pulled steadily the force applied to it will be transmitted from string B to A through the mass M and as tension in A will be greater than in B by Mg (weight of mass M) the string A will break. 
v. If we place a coin on smooth piece of card board covering a glass and strike the card board piece suddenly with a finger. The cardboard slips away and the coin falls into the glass due to inertia of rest.
vi. The dust particles in a durree falls off when it is beaten with a stick. This is because the beating sets the durree in motion whereas the dust particles tend to remain at rest and hence separate.
vii. If the motion of the bus is slow, the inertia of motion will be transmitted to the body of the person uniformly and so the entire body of the person will come in motion with the bus and the person will not experience any jerk.

2. Inertia of motion: It is the inability of a body to change itself its state of uniform motion i.e., a body in uniform motion can neither accelerate nor retard by its own.

EXAMPLES;

i. When a bus or train stops suddenly, a passenger sitting inside tends to fall forward. This is because the lower part of his body comes to rest with the bus or train but the upper part tends to continue its motion due to inertia of motion.
ii. A person jumping out of a moving train may fall forward
iii. An athlete runs a certain distance before taking a long jump. This is because velocity acquired by running is added to velocity of the athlete at the time of jump. Hence he can jump over a longer distance.

3. Inertia of direction: It is the inability of a body to change by itself direction of motion.

EXAMPLES;

i. When a stone tied to one end of a string is whirled and the string breaks suddenly, the stone flies off along the tangent to the circle. This is because the pull in the string was forcing the stone to move in a circle. As soon as the string breaks, the pull vanishes. The stone in a bid to move along the straight line flies off tangentially
ii. The rotating wheel of any vehicle throw out mud, if any, tangentially, due to directional inertia
iii. When a car goes round a curve suddenly, the person sitting inside is thrown outwards.

Note: If the motion of the bus is slow, the inertia of motion will be transmitted to the body of the person uniformly and so the entire body of the person will come in motion with the bus and the person will not experience any jerk

Some Important McQ for Concept

MCQ: A rider on horseback falls when horse starts running all of a sudden because
(a) Rider is taken back
(b) Rider is suddenly afraid of falling
(c) Inertia of rest keeps the upper part of body at rest whereas lower part of the body moves forward with the horse
(d) None of the above

MCQ: When a train stops suddenly, passengers in the running train feel an instant jerk in the forward direction because
(a) The back of seat suddenly pushes the passengers forward
(b) Inertia of rest stops the train and takes the body forward
(c) Upper part of the body continues to be in the state of motion whereas the lower part of the body in contact with seat remains at rest
(d) Nothing can be said due to insufficient data

MCQ: A man getting down a running bus falls forward because
(a) Due to inertia of rest, road is left behind and man reaches forward
(b) Due to inertia of motion upper part of body continues to be in motion in forward direction while feet come to rest as soon as they touch the road
(c) He leans forward as a matter of habit
(d) Of the combined effect of all the three factors stated in (a), (b) and (c)

MCQ: A person sitting in an open car moving at constant velocity throws a ball vertically up into air. The ball falls
(a) Outside the car
(b) In the car ahead of the person
(c) In the car to the side of the person
(d) Exactly in the hand which threw it up
ANSWER: (c) Horizontal velocity of ball and person are same so both will cover equal horizontal distance in a given interval of time and after following the parabolic path the ball falls exactly in the hand which threw it up.

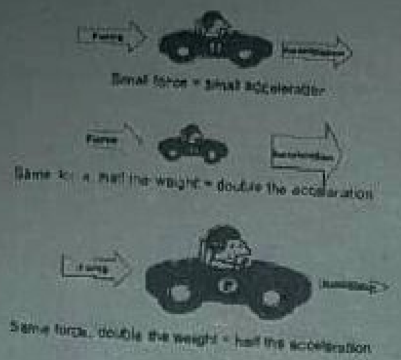
Newton Second Law

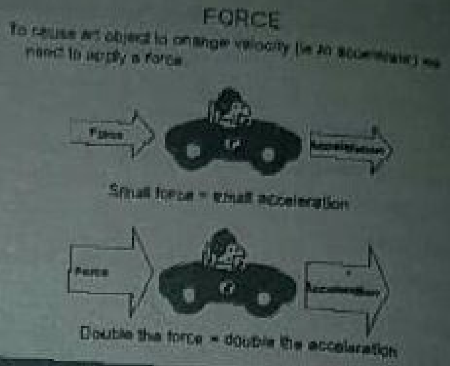
⇒ From Newton's first law, we know that an object with no net force acting on it is in a state of equilibrium.
⇒ We also know that an object experiencing a net force undergoes a change in its velocity.
⇒ But exactly how much does a known force affect the motion of an object?

Force is proportional to mass and acceleration

Imagine pushing a stalled car through a level intersection, as shown in Figure. Because a net force causes an object to accelerate, the speed of the car will increase. When you push the car by yourself, however, the acceleration will be so small that it will take a long time for you to notice an increase in the car's speed. If you get several friends to help you, the net force on the car is much greater, and the car will soon be moving so fast that you will have to run to keep up with it. This change happens because the acceleration of an object is directly proportional to the net force acting on the object. (Note that this is an idealized example that disregards any friction forces that would hinder the motion. In reality, the car accelerates initially. However, when the force exerted by the pushers equals the frictional force, the net force becomes zero, and the car moves at a constant velocity.) Experience reveals that the mass of an object also affects the object's acceleration. A lightweight car accelerates more than a heavy truck if the same force is

applied to both. Thus, it requires less force to accelerate a low-mass object than it does to accelerate a high-mass object at the same rate.





Newton's second law relates force, mass, and acceleration

The relationships between mass, force, and acceleration are quantified in Newton's second law *it states that the acceleration of an object is directly proptional to the net force acting on the object and inversely proportional to the object'smass.*

According to Newton's second law, if equal forces are applied to two objects of different masses, the object with greater mass will experience a smaller acceleration, and the object with less mass will experience a greater acceleration. In equation form, we can state Newton's law as follows:

$$F = ma$$

$$Force = mass \times acceleration$$

Writing the second law: The formula for the second law of motion uses F, m, and a to represent force, mass and acceleration. The way you write the formula depends on what you want to know. Three ways to write this law are summarized below:

Use	If you want to find	If you know
$a = \frac{F}{m}$	*acceleration*	*Force and mass*
$F = ma$	*Force*	*mass and acceleration*
$m = \frac{F}{a}$	*mass*	*Force and acceleration*

Units and The second Law: Using $F = ma$, the units of force (Newton) must be equal to the units of mass (kilograms) multiplied by the units of acceleration (m/sec^2). How is this possible? The answer is that 1 newton is 1 kg.m/sec². The unit "newton" was created to be a shortcut way to write the unit of force. it is much simpler way to say 5 N rather than $5\ kg.m/sec^2$.

According to newton second law **F ∝ m** So the ratio between mass and force is given by: $\frac{F1}{F2} = \frac{m1}{m2}$

Also, we know that acceleration is inversely related with mass $a \propto 1/m$ so the ratio between mass and acceleration is given by: $\frac{a1}{a2} = \frac{m2}{m1}$

And acceleration is directly proportional to force $a \propto f$ so: $\frac{a1}{a2} = \frac{f1}{f2}$

Force

⇒ Concept of force is central to all of physics whether it is classical physics, nuclear physics, quantum physics or any other form of physics

⇒ So, what is force? when we push or pull anybody, we are said to exert force on the body

⇒ Push or pull applied on a body does not exactly define the force in general. We can define force as an influence causing a body at rest or moving with constant velocity to undergo an acceleration

⇒ There are many ways in which one body can exert force on another body
Few examples are given below
(a) Stretched springs exerts force on the bodies attached to its ends
(b) Compressed air in a container exerts force on the walls of the container
(c) Force can be used to deform a flexible object
There are lots of examples you could find looking around yourself

⇒ Force of gravitational attraction exerted by earth is a kind of force that acts on every physical body on the earth and is called the weight of the body

⇒ Mechanical and gravitation forces are not the only forces present infact all the forces in

⇒ In mechanics we will only study about the mechanical and gravitational forces

⇒ Force is a vector quantity/

⇒ Units:

Absolute units: (i) Newton (S.I.) (ii) Dyne (C.G.S)	Gravitational units: (i) Kilogram-force (M.K.S.) (ii) Gram-force (C.G.S)
⇒ Newton: One Newton is that force which produces an acceleration of 1m / s2 in a body of mass $1\ Kilogram;\ 1\ Newton = 1kg\ m/s^2$	⇒ Kilogram-force: It is that force which produces an acceleration of 9.8 m/s² in a body of mass $1\ kg. = 1\ kg - f = 9.81\ Newton$
⇒ Dyne: One dyne is that force which produces an acceleration of 1cm / s2 in a body of mass $1\ gram;\ 1\ Dyne = 1gm\ cm/sec^2$	⇒ Gram-force: It is that force which produces an acceleration of 980cm/s² in a body of mass $1gm. = 1\ gm - f = 980\ Dyne$

⇒ Relation between absolute units of force 1 Newton = 10^5 Dyne

⇒ Relation between gravitational units of force: 1 kg-f = 10^7 gm-f

⇒ $F = ma$ Formula is valid only if force is changing the state of rest or motion and the mass of the body is constant and finite.

⇒ If m is not constant; $\vec{F} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$

⇒ If force and acceleration have three components along x, y and z axis, then;

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k} \text{ and } \vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

From above it is clear that;

$$F_x = ma_x, F_y = ma_y, F_z = ma_z$$

⇒ No force is required to move a body uniformly along a straight line.

$$F = ma \rightarrow F = 0\ (As\ a = 0)$$

⇒ **Positive force:** When force is written without direction then positive force means repulsive while negative force means attractive. Example: Positive force – Force between two similar charges

⇒ **Negative force:** Force between two opposite charges

⇒ **Constant force:** If the direction and magnitude of a force is constant. It is said to be a constant force.

⇒ **Variable or dependent force:**

(i) Time dependent force: In case of impulse or motion of a charged particle in an alternating electric field force is time dependent

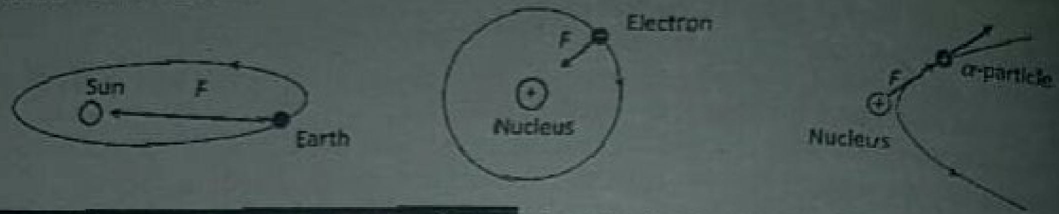
(ii) Position dependent force: Gravitational force between two bodies $G\frac{m_1m_2}{r^2}$

Or Force between two charged particles $\frac{q_1q_2}{4\pi\varepsilon_0r^2}$

⇒ *Velocity dependent force:* Viscous force $(6\pi\eta rv)$

⇒ Force on charged particle in a magnetic field $(qvb\sin\theta)$

⇒ *Central force:* If a position dependent force is always directed towards or away from a fixed point it is said to be central otherwise non-central.
Example: Motion of earth around the sun. Motion of electron in an atom. Scattering of α-particles from a nucleus.

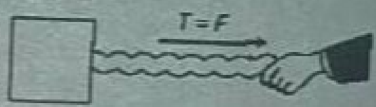


⇒ **Conservative or non-conservative force:** If under the action of a force the work done in a round trip is zero or the work is path independent, the force is said to be conservative otherwise non conservative.
Example: Conservative force: Gravitational force, electric force, elastic force.
Non conservative force: Frictional force, viscous force.

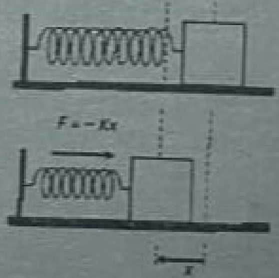
⇒ **Common forces in mechanics:**
(i) Weight: Weight of an object is the force with which earth attracts it. It is also called the force of gravity or the gravitational force.
(ii) Reaction or Normal force: When a body is placed on a rigid surface, the body experiences a force which is perpendicular to the surfaces in contact. Then force is called 'Normal force' or 'Reaction'

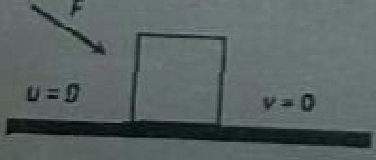


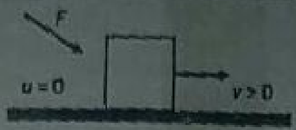
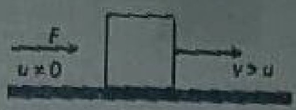
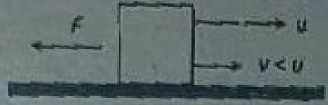
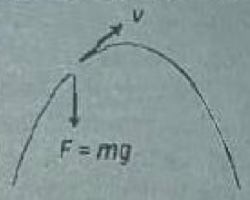
⇒ *Tension:* The force exerted by the end of taut string, rope or chain against pulling (applied) force is called the tension. The direction of tension is so as to pull the body.



⇒ *Spring force:* Every spring resists any attempt to change its length. This resistive force increases with change in length. Spring force is given by $F = -Kx$; where x is the change in length and K is the spring constant (unit N/m).

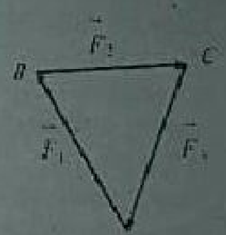


	Body remains at rest. Here force is trying to change the state of rest.

	Body starts moving. Here force changes the state of rest.
	In a small interval of time, force increases the magnitude of speed and direction of motion remains same.
	In a small interval of time, force decreases the magnitude of speed and direction of motion remains same.
	In uniform circular motion only direction of velocity changes, speed remains constant. Force is always perpendicular to velocity.
	In non-uniform circular motion, elliptical, parabolic or hyperbolic motion force acts at an angle to the direction of motion. In all these motions. Both magnitude and direction of velocity changes.

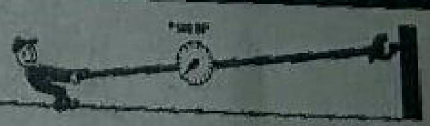
Equilibrium of Concurrent Force:

- ✓ If all the forces working on a body are acting on the same point, then they are said to be concurrent.
- ✓ A body, under the action of concurrent forces, is said to be in equilibrium, when there is no change in the state of rest or of uniform motion along a straight line.
- ✓ The necessary condition for the equilibrium of a body under the action of concurrent forces is that the vector sum of all the forces acting on the body must be zero.
- ✓ Three concurrent forces will be in equilibrium, if they can be represented completely by three sides of a triangle taken in order.



NEWTON'S THIRD LAW

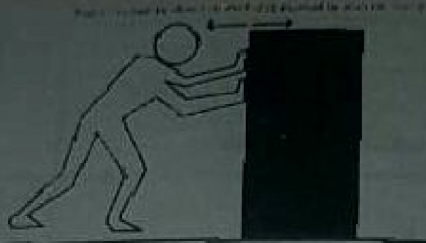
A force is exerted on an object when that object interacts with another object in its environment, when your hand applies a force to a door to push it open, the door simultaneously exerts a force back on your hand.



Forces always exist in pairs

From examples like those discussed in the previous paragraph, Newton recognized that a single isolated force cannot exist. Instead, forces always exist in pairs.





NEWTON'S THIRD LAW:
If two objects interact, the magnitude of force exerted on object 1 by object 2 is equal to the magnitude of force simultaneously exerted on object 2 by object 1, and these two forces are opposite in direction.

⇒ An alternative statement of this law is that for every action, there is an equal and opposite reaction. When two objects interact with one another, the forces that the objects exert on each other are called an action-reaction pair.

⇒ The force that object 1 exerts on object 2 is sometimes called the action force, while the force that object 2 exerts on object 1 is called the reaction force.

⇒ The action force is equal in magnitude and opposite in direction to the reaction force.

⇒ The terms action and reaction sometimes cause confusion because they are used a little differently in physics than they are in everyday speech. In everyday speech, the word reaction is used to refer to something that happens after and in response to an event. In physics, however, the reaction force occurs at exactly the same time as the action force.

Action and reaction forces each act on different objects

One important thing to remember about action-reaction pairs is that each force acts on a different object. Consider the task of driving a nail into wood. To accelerate the nail and drive it into the wood, the hammer exerts a force on the nail. According to Newton's third law, the nail exerts a force on the hammer that is equal to the magnitude of the force that the hammer exerts on the nail. The concept of action-reaction pairs is a common source of confusion because some people assume incorrectly that the equal and opposite forces balance one another and make any change in the state of motion impossible. If the force that the nail exerts on the hammer is equal to the force the hammer exerts on the nail, why doesn't the nail remain at rest? The motion of the nail is affected only by the forces acting on the nail. To determine whether the nail will accelerate, draw a free-body diagram to isolate the forces acting on the nail, as shown in Figure 9. The force of the nail on the hammer is not included in the diagram because it does not act on the nail. According to the diagram, the nail will be driven into the wood because there is a net force acting on the nail. Thus, action-reaction pairs do not imply that the net force on either object is zero. The action-reaction forces are equal and
opposite, but either object may still have a net force acting on it.



The force that the nail exerts on the hammer is equal and opposite to the force that the hammer exerts on the nail.



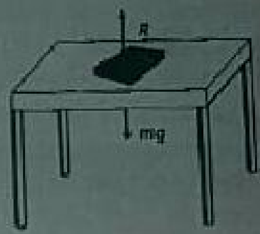
The net force acting on the nail drives the nail into the wood.

MCQ

MCQ: We can derive Newton's
a) second and third laws from the first law b) first and second laws from the third law
c)third and first laws from the second law d)All the three laws are independent of each other
Answer: (c)
MCQ: The law of conservation of momentum is a logical consequence of Newton's
a) first law b) second law c) third law d)all the three laws **Answer: (b)**

Summary; Newton third law

⇒ To every action, there is always an equal (in magnitude) and opposite (in direction) reaction.
⇒ When a body exerts a force on any other body, the second body also exerts an equal and opposite force on the first.
⇒ Forces in nature always occurs in pairs. A single isolated force is not possible.
⇒ Any agent, applying a force also experiences a force of equal magnitude but in opposite direction.
⇒ The force applied by the agent is called 'Action' and the counter force experienced by it is called 'Reaction'.
⇒ Action and reaction never act on the same body. If it were so the total force on a body would have always been zero i.e. the body will always remain in equilibrium.



⇒ If F_{AB} = force exerted on body A by body B (Action) and F_{BA} = force exerted on body B by body A (Reaction)
Then according to Newton's third law of motion $F_{AB} = - F_{BA}$
⇒ Example:
⇒ A book lying on a table exerts a force on the table which is equal to the weight of the book. This is the force of action. The table supports the book, by exerting an equal force on the book. This is the force of reaction. As the system is at rest, net force on it is zero. Therefore force of action and reaction must be equal and opposite.
⇒ Swimming is possible due to third law of motion.
⇒ When a gun is fired, the bullet moves forward (action). The gun recoils backward (reaction)
⇒ Rebounding of rubber ball takes place due to third law of motion.
⇒ While walking a person presses the ground in the backward direction (action) by his feet. The ground pushes the person in forward direction with an equal force (reaction). The component of reaction in horizontal direction makes the person move forward.
⇒ It is difficult to walk on sand or ice.
⇒ Driving a nail into a wooden block without holding the block is difficult. direction in accordance with Newton's third law of motion.

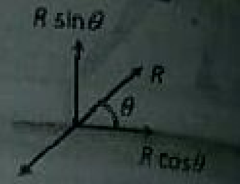
Linear Momentum:

⇒ If you're standing at the bottom of a hill and you're faced with the option of stopping a runaway semi-truck or stopping a runaway bicycle, you'd probably choose to stop the bike, right? The reasoning behind this is that the semi-truck has more **momentum** than the bike.



⇒ Momentum simply means mass in motion.

⇒ The semi-truck has a large momentum because it is very massive, but it also has a large speed, which influences momentum, as well. The bike also has momentum because it has a large speed, but because its mass is less than that of the truck, its momentum is also less.



⇒ It is also measured as the product of the mass of the body and its velocity *i.e., Momentum = mass × velocity.*

⇒ This relationship can be described in an equation: momentum = mass x velocity.

You may remember that **velocity** is speed with direction, so if an object has a large speed, it also has a large velocity.

Our momentum equation can be simplified even more by substituting the words for symbols $p = mv$, where p is momentum, mass (in kg) and v is velocity (in m/s).

⇒ As you can see, if you increase one of the variables on the right side of the equation, either the mass or the velocity, the momentum on the left side must also go up in order to keep both sides equal. If you increase both mass and velocity, the momentum goes up even more.

⇒ This means that an object at rest does not have momentum. The velocity of an object at rest is zero, so there's no movement. In order for an object to have momentum, it must be moving!

⇒ An important thing to note about momentum is that it is a **vector quantity**. This means that it has both magnitude and direction. Velocity is also a vector quantity because it has both of these components, and fortunately for us, the direction of the momentum is the same as the direction of the velocity vector. But in order to fully describe the momentum of an object, you must include its direction – otherwise, it's not a vector.

⇒ The area under F versus t curve gives the change in momentum.

⇒ The slope of p versus t curve gives the force.

⇒ The momentum of a body may be negative.

⇒ A body cannot have momentum without having energy but the body may have energy (i.e., potential energy) without having momentum.

⇒ It depends on frame of reference.

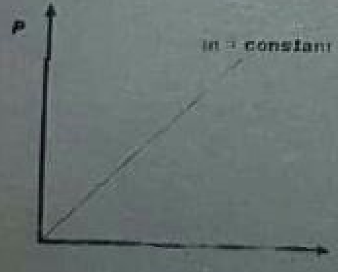
⇒ A meteorite burns in the atmosphere. Its momentum is transferred to air molecules and the earth.

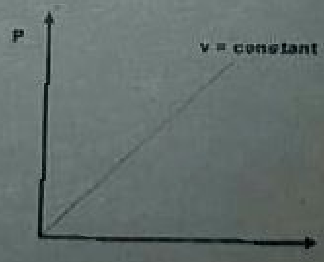
⇒ If two objects of different masses have same momentum, the lighter body possesses greater velocity.

$$p = m_1v_1 = m_2v_2 = constant \Rightarrow \frac{v_1}{v_2} = \frac{m_2}{m_1} \Rightarrow i.e, v \propto \frac{1}{m} \; (As\ p = constant)$$

⇒ For a given body $p \propto v$.

⇒ For different bodies moving with same velocities $p \propto m$.





Concepts:
Faster objects are harder to stop

⇒ Imagine two kids on skateboards are moving toward you. Each has a mass of 40 kilograms. One is moving at one meter per second and the other at 10 meters per second. Which one is harder to stop?

You already learned that inertia comes from mass. That explains why an 80-kilogram skateboarder is harder to stop than a 40-kilogram skateboarder. But how do you account for the fact that a faster skateboarder takes more force to stop than a slower one with the same mass? Momentum The answer is a new quantity called momentum. The momentum of a moving object is its mass multiplied by its velocity. Like inertia, momentum measures a moving object's resistance to changes in its motion. However, momentum includes the effects of speed and direction as well as mass. The symbol p is used to represent momentum.

Momentum

$$P = mv, p \rightarrow Momentum(kg - \frac{m}{sec}), m \rightarrow mass(kg), v \rightarrow velocity(\frac{m}{sec})$$

- **Units of momentum:** The units of momentum are the units of mass multiplied by the units of velocity. When mass is in kilograms and velocity is in meters per second momentum is in kilogram-meters per second (kg-m/sec).

Impulsive forces:

The forces which act on bodies for a short time are called impulsive forces. The examples of impulsive forces are.

i) *A bat hitting the ball*
ii) *The collision of two billiard ball.*
iii) *The firing of a gun.*
iv) *Accident between two cars.*

Note: The impulsive force is constant but vary with time. Therefore, we can't measure impulsive force in such cases we measure the total effect of impulsive forces that are simply called impulse.

Impulse

Force changes momentum

Momentum changes when velocity changes. Since force is what changes velocity, that means that force is also linked to changes in momentum. The relationship with momentum gives us an important new way to look at force. Impulse A change in an object's momentum depends on the net force and also on the amount of time the force is applied. The change in momentum is equal to the net force multiplied by the time the force acts. A change in momentum created by a force exerted over time is called impulse.

$$\vec{F}_{av} = \frac{\overrightarrow{\Delta_p}}{\overrightarrow{\Delta_t}} \quad or$$

$$\overrightarrow{F_{av}}\Delta t = \overrightarrow{\Delta_p} = change\ in\ linear\ momentum.$$

$$\overrightarrow{F_{av}}\Delta t\ this\ product\ is\ called\ impulsive\ force.$$

Hence impulse of a force is the product of average force during the impact and the time for which the impact lasts i.e.

Impulse= $\overrightarrow{F_{av}} \times \Delta t$

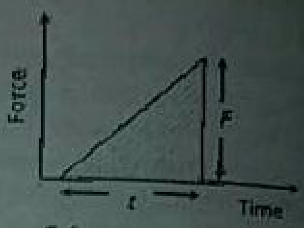
IMPULSE

$$FT = mv_2 - mv_1$$

$F \to Force\ (N), t \to Time\ (sec), FT \to$ Impulse ($N - sec$ or $kg.m/sec$)
$m \to Mass\ (kg), v_2 \to final\ speed\ (\frac{m}{sec}), v_1 \to Initial\ speed\ (\frac{m}{sec})$

Another equation: $J = F\ \Delta t$

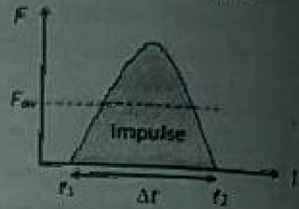
⇒ **Units of impulse:** Notice that the force side of the equation has units of N·sec, while the momentum side has units of momentum, kg m/sec. These are the same units, since 1 N is 1 kg·m/s². Impulse can be correctly expressed either way.



⇒ Impulse is a vector quantity.

⇒ The direction of impulse will same is to that in which direction there is change in linear momentum.

⇒ Dimension of impulse is same is that of linear momentum.



⇒ *Force-time graph:* Impulse is equal to the area under F-t curve. If we plot a graph between force and time, the area under the curve and time axis gives the value of impulse.
J = Area between curve and time axis

$$= \frac{1}{2} \times Base \times Height = \frac{1}{2} Ft$$

⇒ From Newton's second law, $\vec{F} = \frac{dp}{dt} \to \vec{J} = \vec{p_2} - \vec{p_1} = \Delta\vec{p}$ i.e. The impulse of a force is equal to the change in momentum.
This statement is known as Impulse momentum theorem.

Example of impulse linear momentum relationship:

Average impulsive force, $\overrightarrow{F_{av}} = \frac{Change\ in\ linear\ momentum}{Time}$

Therefore, whenever you wish the force of impact to be small, extend the time of impact. On the other hand, if the time of impact is small, the impact of force will be large. There are a large number of practical examples where impulse linear momentum relationship plays an important role. A few cases are being discussed below by way of illustration;

⇒ While catching a ball, a cricket player extends his hands forward so that he has plenty of room to let his hands move backward after making contact with the ball. This extends the time of impact and thus reduces the force of impact.

⇒ A person is better off falling on a wooden floor than a concrete floor. The wooden floor allows for a longer time of impact and, therefore, a lesser force of impact than a concrete floor.

⇒ A person jumping from an elevated position on a floor below bends his knees upon making contact. This extends the time of impact. Therefore, the force of impact is reduced.

⇒ china wares are wrapped in a paper or straw before packing to avoid breakage. This increases the time of impact between various articles during jerks, thereby decreasing the force of impact on the articles.

⇒ while catching a cricket ball, if you do not move your hands away upon contact, you may be hurt. It is because the time of impact will be small so that the impact force will be large.

⇒ When the car goes out of control while driving, you would prefer to hit something soft (say haystack) than something hard (say concrete wall). It is because by hitting haystack so extend your time of impact, thereby reducing the impact force.

MCQ: A ball of mass 10 g hits a hard surface vertically with a speed of 5 m/s and rebounds with the same speed. The ball remains in contact with the surface for 0.01second. Find the average force exerted by the surface on the ball?
Solution: F × t = change in linear momentum → Now change in linear momentum = mv – (-mv) = 2 mv

$F \times t = 2mv$ or $F \times 0.01 = 2 \times \frac{10}{1000} \times 5 \rightarrow F = 10N$

Law of conservation of Linear momentum

⇒ "If a body or a system is isolated then its momentum will be conserved", (In another words there will be no changes in linear momentum).

⇒ Initial momentum and final momentum will be equal.

⇒ **Isolated body:** If there is no external force act on a body then body is said to be isolated.

✓ 100 % isolated body system is not possible.

⇒ **Real isolated Body:** If internal force is very greater then external force then a body or system is said to be real.

✓ For example: Collision of steel ball, Bomb explosion.

⇒ **Definition Explanation:** If external force is equal to zero. Means isolated body then from newton 2nd law we have $F_{ext} = \frac{\Delta P}{\Delta t}$ so

$$0 = \frac{\Delta P}{\Delta t}$$

⇒ **It means**

⇒ $\Delta_P = 0$
⇒ $p_f - p_i = 0$
⇒ $p_f = p_i$
⇒ $mv_f = mv_i$
⇒ $mv = constant$
⇒ v is inversely proportional to mass. $v \propto \frac{1}{m}$

⇒ *Another relation is; as*

$m_1 v_1 = m_2 v_2$ then: $\frac{m1}{m2} = \frac{v2}{v1}$

⇒ For example:
Bomb explosion $p_f = p_i$ Means $\Delta p = 0 \Rightarrow$ momentum conserved

⇒ Recoiling of a gun: For bullet and gun system, the force exerted by trigger will be internal so the momentum of the system remains unaffected



Let $= m_G$ *mass of gun*, m_b *=mass of bullet*
v_g *=velocity of gun*, v_b *=velocity of bullet*

Initial momentum of system = 0

$$Final\ momentum\ of\ a\ system = m_G\vec{v}_G + m_B\vec{v}_B$$

By the law of conservation of linear momentum

$$m_G\vec{v}_G + m_B\vec{v}_B = 0$$

$$so\ recoil\ velocity\ \vec{v}_G = -\frac{m_B}{m_G}\vec{v}_B$$

(a) Here negative sign indicates that the velocity of recoil $\vec{v}_G$ is opposite to the velocity of the bullet.

(b) $v_G = \frac{1}{m_G}$ i.e. higher the mass of gun, lesser the velocity of recoil of gun

(c) While firing the gun must be held tightly to the shoulder, this would save hurting the shoulder because in this condition the body of the shooter and the gun behave as one body. Total mass become large and recoil velocity becomes too small

$$v_G \propto \frac{1}{m_G + m_{man}}$$

CONSERVATION OF MOMENTUM:

$$m_1 v_{1}.i + m_2 v_{2}.i = m_1 v_1.f + m_2 v_2.f$$

TOTAL INITIAL MOMENTUM = TOTAL FINAL MOMENTUM

Momentum is conserved in collisions

In the billiard ball example, we found that the momentum of ball A does not remain constant and the momentum of ball B does not remain constant, but the total momentum of ball A and ball B does remain constant. In general, the total momentum remains constant for a system of objects that interact with one another. In this case, in which the table is assumed to be frictionless, the billiard balls are the only two objects interacting. If a third object exerted a force on either ball A or ball B during the collision, the total momentum of ball A, ball B, and the third object would remain constant. In this book, most conservation-of-momentum problems deal with only two isolated objects. However, when you use conservation of momentum to solve a problem or investigate a situation, it is important to include all objects that are involved in the interaction. Frictional forces—such as the frictional force between the billiard balls and the table—will be disregarded in most conservation-of-momentum.

Momentum is conserved for objects pushing away from each other

Another example of conservation of momentum occurs when two or more interacting objects that initially have no momentum begin moving away from each other. Imagine that you initially stand at rest and then jump up, leaving the ground with a velocity v. Obviously, your momentum is not conserved; before the jump, it was zero, and it became mv as you began to rise. However, the total momentum remains constant if you include Earth in your analysis. The total momentum for you and Earth remains constant. If your momentum after you jump is 60 kg-m/s upward, then Earth must have a corresponding momentum of 60 kg-m/s downward, because total momentum is conserved. However, because Earth has an enormous mass (6×10^{24} kg), its momentum corresponds to a tiny velocity (1×10^{-23} m/s). Imagine two skaters pushing away from each other, as shown in Figure. The skaters are both initially at rest with a momentum of $P_1 i + P_2 i = 0$. When they push away from each other, they move in opposite directions with equal but opposite momentum so that the total final momentum is also zero. $(P_1 f + p_2 f) = 0$

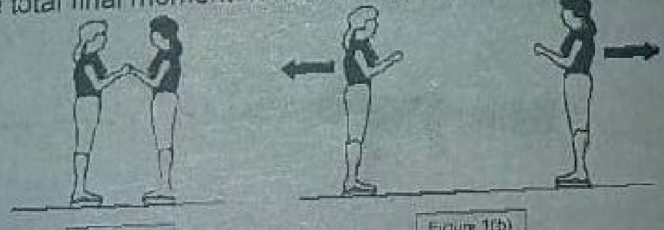


Figure 1(a) — When the skaters stand facing each other, both skaters have zero momentum, so the total momentum of both skaters will be zero

Figure 1(b) — When the skaters push away from each other momentum is equal but opposite, so the total momentum is still zero

Momentum is so fundamental in Newton's mechanics that Newton called it simply "quantity of motion". The symbol for momentum, p, comes from German mathematician Gottfried Leibniz. Leibniz used the term progress to mean "the quantity of motion with which a body proceeds in a certain direction.

Physical meaning of equation p = mv:

Small micro size stone with high velocity can't broke glass. Like that very heavy stone but very low velocity can broke glass. But medium size stone with medium velocity can broke the glass. You can understand physical meaning of momentum like this.

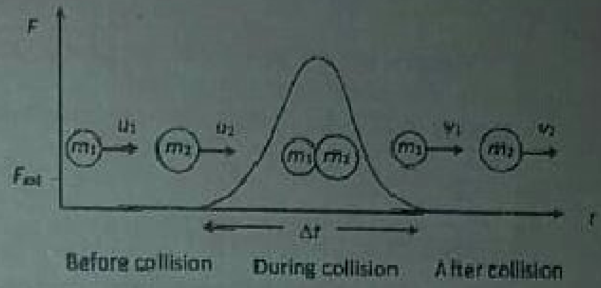
COLLISIONS

⇒ Collision is an isolated event in which a strong force acts between two or more bodies for a short time as a result of which the energy and momentum of the interacting particle change.

⇒ In collision particles may or may not come in real touch e.g. in collision between two billiard balls or a ball and bat there is physical contact

⇒ while in collision of alpha particle by a nucleus (i.e. Rutherford scattering experiment) there is no physical contact.

(1) Stages of collision: There are three distinct identifiable stages in collision, namely, before, during and after. In the before and after stage the interaction forces are zero. Between these two stages, the interaction forces are very large and often the dominating forces governing the motion of bodies. The magnitude of the interacting force is often unknown, therefore, Newton's second law cannot be used, the law of conservation of momentum is useful in relating the initial and final velocities.



(2) Momentum and energy conservation in collision:

(I) MOMENTUM CONSERVATION: In a collision the effect of external forces such as gravity or friction are not taken into account as due to small duration of collision (Δt) average impulsive force responsible for collision is much larger than external force acting on the system and since this impulsive force is 'Internal' therefore the total momentum of system always remains conserved.

(II) ENERGY CONSERVATION: In a collision 'total energy' is also always conserved. Here total energy includes all forms of energy such as mechanical energy, internal energy, excitation energy, radiant energy or even mass energy. ---- -These laws are the fundamental laws of physics and applicable for any type of collision but this law not true for conservation of kinetic energy.

Types of collision: (i) On the basis of conservation of kinetic energy.

PERFECTLY ELASTIC COLLISION	INELASTIC COLLISION	PERFECTLY INELASTIC COLLISION
If in a collision, kinetic energy after collision is equal to kinetic energy before collision, the collision is said to be perfectly elastic.	If in a collision kinetic energy after collision is not equal to kinetic energy before collision, the collision is said to inelastic.	If in a collision two bodies stick together or move with same velocity after the collision, the collision is said to be perfectly inelastic.
$(KE)_{final} = (KE)_{initial}$	Here kinetic energy appears in other forms. In some cases $(KE)_{final} < (KE)_{initial}$ such as when initial KE is converted into internal energy of the product (as heat, elastic or excitation) while in other cases $(KE)_{final} > (KE)_{initial}$ such as when internal energy stored in the colliding particles is released	The term 'perfectly inelastic' does not necessarily mean that all the initial kinetic energy is lost, it implies that the loss in kinetic energy is as large as it can be (Consistent with momentum conservation).
Examples : (1) Collision between atomic particles (2) Bouncing of ball with same velocity after the collision with earth.	Examples: (1) Collision between two billiard balls. (2) Collision between two automobiles on a road. **In fact all majority of**	Example: Collision between a bullet and a block of wood into which it is fired. When the bullet remains embeded in the block.

	collision belong to this category.	

(ii) On the basis of the direction of colliding bodies

Head on or one-dimensional collision	Oblique collision
In a collision if the motion of colliding particles before and after the collision is along the same line the collision is said to be head on or one dimensional	If two particle collision is 'glancing' i.e. such that their directions of motion after collision are not along the initial line of motion, the collision is called oblique. If in oblique collision the particles before and after collision are in same plane, the collision is called 2-dimensional otherwise 3- dimensional.
Example: collision of two gliders on an air track	Example: Collision of billiard balls

Perfectly Elastic Head on Collision:

⇒ Let two bodies of masses m_1 and m_2 moving with initial velocities u_1 and u_2 in the same direction and they collide such that after collision their final velocities are v_1 and v_2 respectively.



Before collision After collision

⇒ According to law of conservation of momentum:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad \text{.......}i$$

$$\rightarrow m_1(u_1 - v_1) = m_2(v_2 - v_1) \quad \text{..........}ii$$

According to law of conservation of kinetic energy

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad \text{.........}iii$$

$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2) \quad \text{.......}iv$$

Dividing equation (iv) by equation (ii)

$$v_1 + u_1 = v_2 + u_2 \quad \text{.....}v$$

$$\rightarrow u_1 - u_2 = v_2 - v_1 \quad \text{.....}vi$$

Relative velocity of approach = Relative velocity of separation

Further from equation (v) we get $v_2 = v_1 + u_1 - u_2$

Substituting this value of v_2 in equation (i) and rearranging we get

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{m_1 + m_2} \quad \text{......}vii$$

Similarly we get

$$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1u_1}{m_1 + m_2} \quad \text{........}viii$$

(1) Special cases of head on elastic collision

(i) If projectile and target are of same mass i.e. $m_1 = m_2$

Since $v_1 = (\frac{m_1-m_2}{m_1+m_2})u_1 + \frac{2m_2u_2}{m_1+m_2}$ and $v_2 = (\frac{m_2-m_1}{m_1+m_2})u_2 + \frac{2m_1u_1}{m_1+m_2}$

Substituting $m_1 = m_2$ we get

$v_1 = u_2$ and $v_2 = u_1$

It means when two bodies of equal masses undergo head on elastic collision, their velocities get interchanged.

Example : Collision of two billiard balls

Sub case: $u_2 = 0$, i.e. target is at rest 0
$v_1 = 0$ and $v_2 = u_1$

(ii) If massive projectile collides with a light target i.e. $m1 >> m2$

Since $v_1 = (\frac{m_1-m_2}{m_1+m_2})u_1 + \frac{2m_2u_2}{m_1+m_2}$ and $v_2 = (\frac{m_2-m_1}{m_1+m_2})u_2 + \frac{2m_1u_1}{m_1+m_2}$

Substituting $m_2 = 0$ we get

$v_1 = u_1$ and $v_2 = 2u_1 - u_2$

Example : Collision of a truck with a cyclist

(iii) If light projectile collides with a very heavy target i.e. $m_1 << m_2$

Since $v_1 = (\frac{m_1-m_2}{m_1+m_2})u_1 + \frac{2m_2u_2}{m_1+m_2}$ and $v_2 = (\frac{m_2-m_1}{m_1+m_2})u_2 + \frac{2m_1u_1}{m_1+m_2}$

Substituting $m1 = 0$, we get

$v_1 = -u_1 + 2u_2$ and $v_2 = u_2$

Example : Collision of a ball with a massive wall.

Sub case : i.e. target is at rest
$v_1 = u_1$ and $v_2 = 2u_1$ $u2 = 0$

(2) Kinetic energy transfer during head on elastic collision

Kinetic energy of projectile before collision $k_i = \frac{1}{2}m_1u_1^2$

Kinetic energy of projectile after collision $k_f = \frac{1}{2}m_1v_1^2$

Kinetic energy transferred from projectile to target ΔK = decrease in kinetic energy in projectile

$$\Delta k = \frac{1}{2}m_1u_1^2 - \frac{1}{2}m_1v_1^2 = \frac{1}{2}m_1(u_1^2 - v_1^2)$$

Fractional decrease in kinetic energy $\frac{\Delta k}{k} = \frac{\frac{1}{2}m_1(u_1^2-v_1^2)}{\frac{1}{2}m_1u_1^2} = 1 - (\frac{v_1}{u_1})^2$ i

We can substitute the value of v1 from the equation $v_1 = (\frac{m_1-m_2}{m_1+m_2})u_1 + \frac{2m_2u_2}{m_1+m_2}$

If the target is at rest i.e. $u2 = 0$ then $v_1 = (\frac{m_1-m_2}{m_1+m_2})u_1$

From equation (i) $\frac{\Delta k}{k} = 1 - (\frac{m_1-m_2}{m_1+m_2})^2$ ii

$$or \quad \frac{\Delta k}{k} = \frac{4m_1m_2}{(m_1+m_2)^2} \text{ .... iii}$$

$$Or \quad \frac{\Delta k}{k} = \frac{4m_1m_2}{(m_1-m_2)^2+4m_1m_2} \text{ .... iv}$$

Note: Greater the difference in masses less will be transfer of kinetic energy and vice versa Transfer of kinetic energy will be maximum when the difference in masses is minimum

$$i.e.,\ m_1 - m_2 = 0 \quad or \quad m_1 = m_2 \quad then, \quad \frac{\Delta k}{k} = -1 = 100\%$$

So, the transfer of kinetic energy in head on elastic collision (when target is at rest) is maximum when the masses of particles are equal i.e. mass ratio is 1 and the transfer of kinetic energy is 100%

⇒ If $m_2 = nm_1$ then from equation (iii) we get $\frac{\Delta k}{k} = \frac{4n}{(1+n)^2}$

- Kinetic energy retained by the projectile

$$(\frac{\Delta k}{k})_{Retained} = 1 - kinetic\ energy\ transferred\ by\ projectile.$$

$$\left(\frac{\Delta k}{k}\right)_{Retained} = 1 - \left[1 - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2\right] = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$$

(3) Velocity, momentum and kinetic energy of stationary target after head on elastic collision

(i) Velocity of target: We know,

$$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1u_1}{m_1 + m_2}$$

$$v_2 = \frac{2m_1u_1}{m_1 + m_2} = \frac{2u_1}{1 + m_2/m_1}$$

$$[as\ u_2 = 0\ and\ let\ \frac{m_2}{m_1} = n]$$



$$v_2 = \frac{2u_1}{1 + n}$$

(ii) Momentum of target:

$$p_2 = m_2v_2 = \frac{2nm_1u_1}{1 + n} \quad [as\ m_2 = m_1n\ and\ v_2 = \frac{2u_1}{1 + n}]$$

$$p_2 = \frac{2m_1u_1}{1 + (1/n)}$$

(iii) Kinetic energy of target:

$$k_2 = \frac{1}{2}m_2v_2^2 = \frac{1}{2}nm_1\left(\frac{2u_1}{1 + n}\right)^2 = \frac{2m_1u_1^2n}{(1 + n)^2}$$

$$= \frac{4(k_1)n}{(1 - n)^2 + 4n} \quad [As \quad k_1 = \frac{1}{2}m_1u_1^2]$$

(iv) Relation between masses for maximum velocity, momentum and kinetic energy

Velocity	$v_2 = \frac{2u_1}{1 + n}$	For v_2 to be maximum n must be minimum. i.e $n = \frac{m_2}{m_1} \to 0$ $m_2 \ll m_1$	Target should be very light
Momentum	$p_2 = \frac{2m_1u_1}{(1 + 1/n)}$	For P_2 to be maximum, (1/n) must be minimum or n must be maximum. i.e ....$n = \frac{m_2}{m_1} \to \infty$ $m_2 \gg m_1$	Target should be massive
Kinetic energy	$k_2 = \frac{4k_1n}{(1 - n)^2 + 4n}$	For K_2 to be maximum $(1 - n)^2$ must be minimum. $1 - n = 0 \Rightarrow n = 1 = \frac{m_2}{m_1}$ $m_2 = m_1$	Target and projectile should be of equal mass

Perfectly Elastic Oblique Collision

Let two bodies moving as shown in figure. By law of conservation of momentum

Along x-axis,

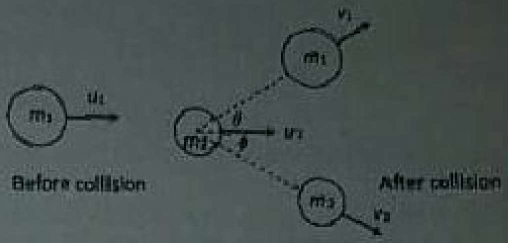
$$m_1u_1 + m_2u_2 = m_1v_1\cos\theta + m_2v_2\cos\varphi \ldots\ldots i$$

Along y-axis,

$$0 = m_1v_1\sin\theta - m_2v_2\sin\varphi \ldots\ldots ii$$

By law of conservation of kinetic energy

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \ldots\ldots iii$$



In case of oblique collision, it becomes difficult to solve problem when some experimental data are provided as in these situations more unknown variables are involved than equations formed.

Special condition:

If $m_1 = m_2$ and $u_2 = 0$ substituting these values in equation (i), (ii) and (iii) we get

$$u_1 = v_1\cos\theta + v_2\cos\varphi \ldots\ldots iv$$

$$0 = v_1\sin\theta - v_2\sin\varphi \ldots\ldots v$$

And;

$$u_1^2 = v_1^2 + v_2^2 \ldots\ldots vi$$

Squaring (iv) and (v) and adding we get

$$u_1^2 = v_1^2 + v_2^2 + 2v_1u_2\cos(\theta + \varphi) \ldots\ldots vii$$

Using (vi) and (vii) we get $\cos(\theta + \varphi) = 0$

$$\theta + 4 = \pi / 2$$

i.e. after perfectly elastic oblique collision of two bodies of equal masses (if the second body is at rest), the scattering angle $\theta + \varphi$ would be 90°.

Head on Inelastic Collision

(1) Velocity after collision: Let two bodies A and B collide in elastically and coefficient of restitution is e. Where;

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approacH}}$$

$$\Rightarrow v_2 - v_1 = e(u_1 - u_2)$$

$$v_2 = v_1 + e(u_1 - u_2) \ldots\ldots i$$

From the law of conservation of linear momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \ldots\ldots ii$$

By solving (i) and (ii) we get

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 + \left(\frac{(1+e)m_2}{m_1 + m_2}\right)u_2$$

Samilarly;

$$v_2 = \left(\frac{(1+e)m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - em_1}{m_1 + m_2}\right)u_2$$

By substituting e = 1, we get the value of v_1 and u_2 for perfectly elastic head on collision.

MCQ: The rate of change of momentum is called______?

(a) Force (b) Impulse (c) Work (d) Displacement

Solution: We know that F = ma → Acceleration = rate of change of velocity→ so putting value in equation F = ma → it becomes→ F = mv/t → F = p/t→ The rate of change of momentum is force.

Projectile Motion:

1. Introduction:

⇒ A hunter aims his gun and fires a bullet directly towards a monkey sitting on a distant tree. If the monkey remains in his position, he will be safe but at the instant the bullet leaves the barrel of gun.

⇒ if the monkey drops from the tree, the bullet will hit the monkey because the bullet will not follow the linear path.



⇒ The path of motion of a bullet will be parabolic and this motion of bullet is defined as projectile motion.

⇒ If the force acting on a particle is oblique with initial velocity then the motion of particle is called projectile motion.

Projectile:

A body which is in flight through the atmosphere but is not being propelled by any fuel is called projectile.

Example: (i) A bomb released from an airplane in level flight
(ii) A bullet fired from a gun
(iii) An arrow released from bow
(iv) A Javelin thrown by an athlete

Assumptions of Projectile Motion:

(1) There is no resistance due to air
(2) The effect due to curvature of earth is negligible.
(3) The effect due to rotation of earth is negligible.
(4) For all points of the trajectory, the acceleration due to gravity 'g' is constant in magnitude and direction.

Principles of Physical Independence of Motions:

(1) The motion of a projectile is a two-dimensional motion. So, it can be discussed in two parts Horizontal motion and vertical motion. These two motions take place independent of each other This is called the principle of physical independence of motions.
(2) The velocity of the particle can be resolved into two mutually perpendicular components. Horizontal component and vertical component.
(3) The horizontal component remains unchanged throughout the flight. The force of gravity continuously affects the vertical component.
(4) The horizontal motion is a uniform motion and the vertical motion is a uniformly accelerated retarded motion.

Instantaneous velocity v_i: In projectile motion, vertical component of velocity changes but horizontal component of velocity remains always constant.

Example: When a man jumps over the hurdle leaving behind its skateboard then vertical component of his velocity is changing, but not the horizontal component, which matches with the skateboard velocity.

As a result, the skateboard stays underneath him, allowing him to land on it.

Let v_i be the instantaneous velocity of projectile at time t direction of this velocity is along the tangent to the trajectory at point P.

$$\vec{v_i} = v_x\hat{i} + v_y\hat{j} \Rightarrow v_i = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2\cos^2\theta + (u\,sin\theta - gt)^2}$$

$$v_i = \sqrt{u^2 + g^2t^2 - 2ugtsin\theta}$$

Direction of instantaneous velocity $tan\alpha = \frac{v_y}{v_x} = \frac{usin\theta - gt}{ucos\theta}$ or $\alpha = \tan^{-1}|tan\theta - \frac{gt}{u}sec\theta|$

(4) Change in velocity: Initial velocity (at projection point) $\vec{u}_i = ucos\theta\hat{i} + usin\theta\hat{j}$

Final velocity (at highest point) $\vec{u}_f = ucos\theta\hat{i} + 0\hat{j}$

✓ Change in velocity (Between projection point and highest point) $\Delta u = \vec{u}_f - \vec{u}_i = -usin\theta\hat{j}$

When body reaches the ground after completing its motion then final velocity

$$\vec{u}_f = u\cos\theta\hat{i} - u\sin\theta\hat{j}$$

✓ Change in velocity (Between complete projectile motions)

$$\Delta u = u_f - u_i = -2u\sin\theta\hat{i}$$

(5) Change in momentum: Simply by the multiplication of mass in the above expression of velocity

✓ Change in momentum (Between projection point and highest point)

$$\Delta p = \vec{p}_f - \vec{p}_i = -mu\sin\theta\hat{j}$$

✓ Change in momentum (For the complete projectile motion)

$$\Delta p = \vec{p}_f - \vec{p}_i = -2mu\sin\theta\hat{j}$$

(6) Angular momentum: Angular momentum of projectile at highest point of trajectory about the point of projection is given by

$$L = mvr \; |Here\ r = H = \frac{u^2\sin^2\theta}{2g}|$$

$$\therefore L = mu\cos\theta\frac{u^2\sin^2\theta}{2g} = \frac{mu^3\cos\theta\sin^2\theta}{2g}$$

(7) Time of flight: The total time taken by the projectile to go up and come down to the same level from which it was projected is called time of flight.

For vertical upward motion $0 = u\sin\theta - gt \Rightarrow t = (u\sin\theta/g)$

Now as time taken to go up is equal to the time taken to come down so

Time of flight $T = 2t = \frac{2u\sin\theta}{g}$

✓ Time of flight can also be expressed as: $T = \frac{2u_y}{g}$ (where u_y is the vertical component of initial velocity).

✓ For complementary angles of projection θ and $90° - \theta$

✓ Ratio of time of flight $= \frac{T_1}{T_2} = \frac{\frac{2u\sin\theta}{g}}{\frac{2u\sin(90-\theta)}{g}} = \tan\theta \Rightarrow \frac{T_1}{T_2} = \tan\theta$

✓ Multiplication of time of flight $= T_1T_2 = \frac{2u\sin\theta}{g}\frac{2u\cos\theta}{g} \Rightarrow T_1T_2 = \frac{2R}{g}$

✓ If t_1 is the time taken by projectile to rise up to point p and t_2 is the time taken in falling from point p to ground level then $t_1 + t_2 = \frac{2u\sin\theta}{g} = time\ of\ flight$

$$or \quad u\sin\theta = \frac{g(t_1+t_2)}{2}$$

and height of the point p is given by $h = u\sin\theta t_1 - \frac{1}{2}gt_1^2$

$$h = g\frac{(t_1+t_2)}{2}t_1 - \frac{1}{2}gt_1^2 \Rightarrow by\ solving\ h = \frac{gt_1t_2}{2}$$

✓ If B and C are at the same level on trajectory and the time difference between these two points is t_1, similarly A and D are also at the same level and the time difference between these two positions is t_2 then

$$t_2^2 - t_1^2 = \frac{8h}{g}$$

(8) Horizontal range: It is the horizontal distance travelled by a body during the time of flight. So by using second equation of motion

$$R = u\cos\theta \times T = u\cos\theta \times \left(\frac{2u\sin\theta}{g}\right) = \frac{u^2\sin2\theta}{g}$$

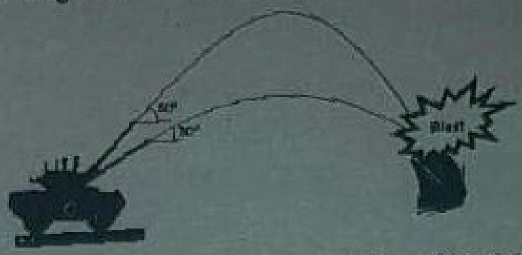
$$R = \frac{u^2\sin2\theta}{g}$$

- ✓ Range of projectile can also be expressed as:

$$R = ucos\theta \times T = ucos\theta \frac{2usin\theta}{g} = \frac{2ucos\theta usin\theta}{g} = \frac{2u_x u_y}{g}$$

(Where u_x and u_y are the horizontal and vertical component of initial velocity)

- ✓ If angle of projection is changed from θ to θ' = (90 − θ) then range remains unchanged



$$R' = \frac{u^2 sin2\theta'}{g} = \frac{u^2 \sin[2(90^\circ - \theta)]}{g} = \frac{u^2 sin2\theta}{g} = R$$

- ✓ So a projectile has same range at angles of projection θ and (90 − θ), though time of flight, maximum height and trajectories are different.
- ✓ These angles θ and 90° − θ are called complementary angles of projection and for complementary angles of projection ratio of range

$$\frac{R_1}{R_2} = \frac{u^2 sin2\theta/g}{u^2 \sin[\frac{2(90^\circ - \theta]}{g}} = 1 \Rightarrow \frac{R_1}{R_2} = 1$$

- ✓ For angle of projection $\theta_1 = (45 - \alpha)$ and $\theta_2 = (45 + \alpha)$, range will be same and equal to

$$u^2 cos\frac{2\alpha}{g}.$$

θ_1 and θ_2 are also the complementary angles.

- ✓ **Maximum range:** For range to be maximum

$$\frac{dR}{d\theta} = 0 \Rightarrow \frac{d}{d\theta}|\frac{u^2 sin2\theta}{g}| = 0$$

$$cos\,2\theta = 0\ i.e.\ 2\theta = 90^\circ \Rightarrow \theta = 45^\circ \text{ and } R_{max} = [\frac{u^2}{g}]$$

i.e., a projectile will have maximum range when it is projected at an angle of 45° to the horizontal and the maximum range will be (u^2/g).

When the range is maximum, the height H reached by the projectile

$$H = \frac{u^2 \sin^2\theta}{2g} = \frac{u^2 \sin^2 45^\circ}{2g} = \frac{u^2}{4g} = \frac{R_{max}}{4}$$

i.e., if a person can throw a projectile to a maximum distance R_{max}, The maximum height to which it will rise is

$$[\frac{R_{max}}{4}]$$

- ✓ Relation between horizontal range and maximum height:

$$R = \frac{u^2 sin2\theta}{g} \text{ and } H = \frac{u^2 \sin^2\theta}{2g}$$

$$\therefore \frac{R}{H} = \frac{\frac{u^2 sin2\theta}{g}}{\frac{u^2 \sin^2\theta}{2g}} = 4cot\theta \Rightarrow R = 4H\,cot\theta$$

- ✓ If in case of projectile motion range R is n times the maximum height H

$$i.e.\ R = nH \Rightarrow \frac{u^2 sin2\theta}{g} = n\frac{u^2 \sin^2\theta}{2g} = tan\theta = |\frac{4}{n}| \text{ or } \theta = tan^{-1}|\frac{4}{n}|$$

The angle of projection is given by $\theta = \tan^{-1}\left|\frac{4}{n}\right|$

✓ Note: *If* $R = H$ *then* $\theta = \tan^{-1}(4)$ *or* $\theta = 76°$.

✓ If R = 4H then $\theta = \tan^{-1}(1)$ *or* $\theta = 45°$.

(9) Maximum height: It is the maximum height from the point of projection, a projectile can reach.

So, by using $v^2 = u^2 + 2as$

$$0 = (u\sin\theta)^2 - 2gH \Rightarrow H = \frac{u^2\sin^2\theta}{2g}$$

✓ Maximum height can also be expressed as

$$H = \frac{u_y^2}{2g}\,(\text{where } u_y \text{ is the vertical component of initil velocity})$$

✓ $H_{max} = \frac{u^2}{2g}$ (*where* $\sin^2\theta = max = 1$ *i.e.*, $\theta = 90°$)

✓ i.e., for maximum height body should be projected vertically upward. So it falls back to the point of projection after reaching the maximum height.

For complementary angles of projection θ and 90° – θ

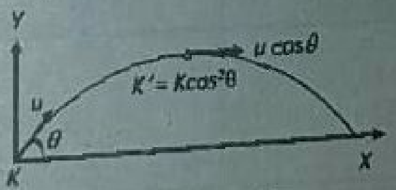
✓ Ratio of maximum height $= \frac{H_1}{H_2} = \frac{\frac{u^2\sin^2\theta}{2g}}{\frac{u^2\sin^2(90-\theta)}{2g}} = \frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta \quad \therefore \frac{H_1}{H_2} = \theta$

⇒ Note that the motion of a projectile relative to another projectile is a straight line.

(12) Energy of projectile: When a projectile moves upward its kinetic energy decreases, potential energy increases but the total energy always remain constant.

If a body is projected with initial kinetic energy K(=1/2 mu²), with angle of projection θ with the horizontal then at the highest point of trajectory

(i) **Kinetic energy** $= \frac{1}{2}m(u\cos\theta)^2 = \frac{1}{2}mu^2\cos^2\theta$



$$\therefore k' = k\cos^2\theta$$

(ii) **Potential energy** $= mgH = \frac{u^2\sin^2\theta}{2g} = \frac{1}{2}mu^2\sin^2\theta \qquad (As\ H = \frac{u^2\sin^2\theta}{2g})$

(iii) **Total energy** = Kinetic energy + Potential energy $= \frac{1}{2}mu^2\cos^2\theta\,\frac{1}{2}mu^2\sin^2\theta$

$$= \frac{1}{2}mu^2 = \text{Energy at the point of projection}$$

This is in accordance with the law of conservation of energy

Formula Sheet

S/N	QUANTIY	EQUATION
1)	*speed*	$\frac{distance}{time} = \frac{s}{t}$
2)	*velocity*	$\frac{displacement}{time} = \frac{d}{t}$
3)	*acceleration*	$a = \frac{velocity}{time} = \frac{v}{t}$
4)	*slope*	$slope = tan\theta = \frac{vvertical\ paraameter}{Horizental\ parameter}$
5)	*Equations of motion*	$v_f = v_i + at$ (1st) $s = v_it + \frac{1}{2}at^2$ (2nd) $2as = v_f^2 - v_i^2$ (3rd) For free fall motion replace a by 'g' in above all equation
6)	*Some other equation For Distannce covered in and after n − second*	Distance covered after n-second: $h = \frac{1}{2}gt^2$ Distance covered in n-second: $h = \frac{1}{2}g(2n-1)$ if $\vec{a}$ is uniform: $S_n = (2n-1)x$ If $\vec{a}$ is non-uniform: $S_n = (2n-1)(\frac{a}{2})x$
7)	*Newton 1st law of motion*	$if\ F_{net} = 0\ then\ a = 0$
8)	*Newton 2nd law*	$F = ma \Rightarrow a = \frac{f}{m}$ $\frac{F_1}{F_2} = \frac{m_1}{m_2}$ or $\frac{a_1}{a_2} = \frac{m_2}{m_1}$
9)	*Newton 3rd law*	$F_{12} = -F_{21}$
10)	*Linear momentum*	$P = mv \Rightarrow v = \frac{p}{m}$
11)	*Newton 2nd law in terms of* $\vec{P}$	$F = \frac{\Delta p}{\Delta t} = \frac{p_f - p_i}{\Delta t}$
12)	*Impulse*	$I = J = F\Delta t \rightarrow F \propto \frac{1}{\Delta t}$
13)	*For free fall motion*	$Weight = \frac{\Delta p}{\Delta t}$
14)	*Conservation of linear momentum*	$F_{net} = \frac{\Delta p}{\Delta t} \Rightarrow 0 = \frac{\Delta p}{\Delta t}$ $\Delta p = 0$ $p_f - p_i = 0$ $m_f v_f = m_i v_i$ $mv = constant\ \ so\ m_1v_1 = m_2v_2$ $\frac{m_1}{m_2} = \frac{v_2}{v_1} \Rightarrow m \propto \frac{1}{v}$
15)	*Elastic collision*	$k.e\ conserved\ or\ \Delta k.e = 0$

		$P = conserved$ or $\Delta p = 0$ $T.E = conserved$
16)	InElastic collision	$K.E \neq constant$ or $\Delta K.E \neq 0$ $P = conserved$ $T.E = conserved$
17)	Law of recisssion	$v_{rel} = -v'_{rel}$ Relative speed of approach before collision = Relative speed of recession after collision -ve sign show moving away
18)	Perfectly elastic collision	$v'_1 = (\frac{m_1 - m_2}{m + m_1})v_1 + (\frac{2m_2}{m_1 + m_2}) \times v_2$ $v'_2 = (\frac{m_2 - m_1}{m_1 + m_2})v_2 + (\frac{2m_1}{m_1 + m_2})v_1$
19)	Important cases	If target particle is at rest; $v_2 = 0$ $v'_1 = (\frac{m_1 - m_2}{m + m_1})v_1$ $v'_2 = (\frac{2m_1}{m_1 + m_2})v_1$
20)	Some other cases	If $m_1 = m_2 = m$ $v'_1 \approx 0$ and $v'_2 \approx v_1$ If $m_1 \gg m_2$ $v'_1 \approx v_1$ and $v'_2 = 2v_1$ If $m_2 \gg m_1$ $v'_1 \approx -v_1$ and $v'_2 = 0$ If $m_1 = m_2 = m$ and $v_1 \neq 0$ and $v_2 \neq 0$ $v'_1 = v_2$ and $v'_2 = v_1$
1)	Projectile motion	Horizental force; $F_x = 0, a_x = 0, v_x = constant$ Vertical force; $F_y = constant$ $F_y \neq 0$ and $a_y \neq 0$ and $a_y = -g$ $v_y = different$
2)		Instantaneous horizontal component of v: $v_x = v_o cos\theta$ Instantaneous horizontal distance travelled $x = (v_o cos\theta)t$ Instantaneous vertical component of velocity: $\Delta v_y = v_o sin\theta - gt$ Instantaneous vertical distance(height) travelled: $y = (v_o sin\theta)t - \frac{1}{2}gt^2$ Instantaneous velocity of projectile: $V = \sqrt{v_x^2 + v_y^2}$ or $v = \sqrt{(v_o cos\theta)^2 + (v_o y - gt)^2}$ At point of projection: $v_y = maximum$ $v_p = v_t$ and so $P - max$ and $K.e - max$
		At point of summit:

$$v = minimum$$

$$v_{summit} = v_x \ and \ so \ K.e - min \ and \ P - min$$

Kinetic energy of projection:

$$K.E_p = K.E_t = \frac{1}{2}mv_o^2$$

Kinetic energy at summit:

$$K.E_s = \frac{1}{2}mv_x^2$$

Momentum at projection:

$$P_p = P_t = mv_o$$

Momentum at summit:

$$P_{summit} = P_p cos\theta$$

Instantaneous position of projectile:

$$r = \sqrt{x^2 + y^2}$$

For projection angle:

$$tan\theta = \frac{y}{x} \Rightarrow \theta = \tan^{-1}\left(\frac{y}{x}\right)$$

Maximum height of projectile:

$$H_{max} = \frac{(v_o^2 \sin^2\theta)}{2g}$$

Time of flight:

$$T = \frac{2v_o sin\theta}{g}$$

Relation between T and H_{max}:

$$H = \frac{gt^8}{8} \Rightarrow T = \frac{8}{g}H_{max}$$

Range of projectile:

$$R = \frac{v_o^2}{g}(sin2\theta)$$

Maximum horizontal distance:

$$R_{max} = \frac{v_o^2}{g}$$

Range will be same for 2 complementary angles

$$R = H \ at \ \theta = 76°$$

$$R \leq 4H$$

$$R = 4H \ at \ \theta = 45° \ maximum \ range$$

Some Important Points and Tricks:

⇒ While the object is rising:
The displacement of the object is up (negative).
- The velocity of the object is up (negative).
- The acceleration of the object is *down* (positive).

⇒ The instant the object reaches its maximum height:
- The displacement of the object is zero.
- The velocity of the object is zero.
- The acceleration of the object is down (positive).

⇒ While the object is falling:
- The displacement of the object is down (positive).
- The velocity of the object is down (positive).
- The acceleration of the object is down (positive).

⇒ All objects in free fall near the earth are accelerated toward the earth (down) at a rate of 9.8 m/s^2.

⇒ Other interesting facts about objects that are thrown straight up near the earth and return to the same spot are:
- Time up = time down.
- Displacement up = displacement down.
- Velocity up = velocity down.

⇒ A particle is moving with a constant speed along a straight-line path. A force is not required to Keep it moving with uniform velocity because Particle will move with uniform velocity due to inertia.

⇒ An object will continue moving uniformly until the resultant force on it is zero.

⇒ When we jump out of a boat standing in water it moves in backward Because for every action there is an equal and opposite reaction takes place.

⇒ A man is at rest in the middle of a pond on perfectly smooth ice. He can get himself to the shore by making use of Newton's 3rd law.

⇒ Newton's third law of motion leads to the law of conservation of momentum.

⇒ A man is standing on a balance and his weight is measured. If he takes a step in the left side, then weight remains same.

⇒ A cold soft drink is kept on the balance. When the cap is open, then the weight First increases then decreases because Gas will come out with sufficient speed in forward direction, so reaction of this forward force will change the reading of the spring balance.

⇒ When a horse pulls a wagon, the force that causes the horse to move forward is the force the ground exerts on it.

⇒ A student attempts to pull himself up by tugging on his hair. He will not succeed as the force applied is internal to the system. As by an internal force momentum of the system cannot be changed.

⇒ A man is standing at the Centre of frictionless pond of ice. How can he get himself to the shore by spitting horizontally?

⇒ The momentum is most closely related to impulse (not to FORCE, POWER OR K.E)

⇒ A body, whose momentum is constant, must have constant velocity because For a given mass $P \propto v$. If the momentum is constant then it's velocity must have constant.

MULTIPLE CHOICE QUESTIONS

1) A bomb is dropped from an aero plane moving horizontally at constant speed. If air resistance is taken in to consideration, then the bomb?
a) falls on earth exactly below the aero plane
b) falls on the earth exactly behind the aero plane
c) falls on the earth ahead of aero plane
d) flies with the aero plane

2) A stone is thrown with an initial speed of 4.9 m/s from a bridge in vertically upward direction. It falls down in water after 2 sec. The height of the bridge is ?
a) 24.7 m b) 19.8 m
c) 9.8 m d) 4.9 m

3) A missile moves is fired for maximum range with initial velocity 20 m/s. If g=10 m/s^2, the range of the missile is_____?
a) 40 m b) 50m
c) 60 m d) 20 m

4) If a body A of mass M is thrown with velocity 'a' at an angle of 30° to the horizontal and another body B of same mass is thrown with same speed at angle of 60° to the horizontal the ratio of horizontal range of A and B will be_____?
a) 1 : 3 b) 1:1
c) $1:\sqrt{3}$ d) $\sqrt{3}:1$

5) From a 10 m high building a stone 'A' is dropped and simultaneously another identical stone 'B' is thrown horizontally with an initial speed of 5 m/s, which one of the following statement is true?
a) It is not possible to calculate which

one of the two stones ill reach the ground first
b) Both the stones ('A' and 'B') will reach the ground simultaneously
c)'A' stone reach the ground earlier than 'B'
d)'B' stone reach the ground earlier than 'A'

6) Two projectiles are fired from the same point with same speed at an angle of projection 60° and 30° respectively. Which one of the following is true?
a) Their maximum height will be same
b) Their range will be same
c)Their landing velocity will be same
d)There time of flight will be same

7) A ball whose kinetic energy is E, is thrown at an angle of 45° with the horizontal. It's kinetic energy at the highest point of its flight will be?
a) E b) E/2
c) E/√2 d) 0

8) A particle moves with a velocity given by vector v=(6i -4j +3k) m/s, under the influence of a constant force F=20i + 15j - 5k N. The instantaneous power applied to the particle is?
a) 45 J/s b) 35 J/s
c) 25 J/s d) 195 J/s

9) A bullet is fired from a gun with speed of 1000 m/s in order to fit a target 100 m away. At what height above the target should the gun be aimed? (resistance of air is negligible and $g=10\ m/s^2$?
a) 5cm b) 10cm
c)15cm d)20cm

10) for angle of projection of a particle (45° - θ) and (45° + θ), the horizontal ranges described by the projectile are in the ratio?
a) 1 : 3 b) 1 : 2
c)2 : 1 d)1 : 1

11) The maximum range of a gun of horizontal terrain is 16 km. If $g=10\ m/s^2$, then muzzle velocity of a shell must be?
a) 160 m/s b) 200√2 m/s
c)400 m/s d)800 m/s

12) A body constrained to move in y-direction , is subjected to force given by F=(-2i + 15j + 6k) N. What is the work done by this force in moving the body through a distance of 10m along y-axis?
a)190 J b) 160 J
c)150 J d)20 J

13) A body of 3kg moves in the XY plane under the action of force given by 6ti + 4tj . Assuming that the body is at rest at time t=0, the velocity of the body at t=3s is?
a)6i + 6j b) 18i + 6j
c)18i + 12j d)12i + 18j

14) for ordinary terrestrial experiments, the observer in an inertial frame in the following cases is
a) a child revolving in a giant wheel
b) a driver in a sports car moving with constant speed of 200 km/h on a straight road
c) the pilot of an aero-plane which is taking off
d) a cyclist negotiating a sharp curve

15) At the uppermost point of projectile, its velocity and acceleration are at an angle of?
a) 180° b) 90°
c)60° d)150°

16) If R and H represent the horizontal range and the maximum height achieved by a projectile then which of the relation exists?
a) H/R=4cotθ b) R/H=4cotθ
c)H/R=4tanθ d)R/H=4tanθ

17) A projectile can have the same range R for two angle of projection. If t1 and t2 be the time of flights in the two cases, then the product of two time of flights is proportional to?
a) $1/R^2$ b) R^2
c) R d) 1/R

18) A ball whose kinetic energy is E, projected at an angle of 60° to the horizontal. The kinetic energy of the ball at the highest point of its flight will be?
a) E b) E/ √2
c)E/4 d)zero

19) A projectile can have the same range 'R' for two angles of projection. If T_1 and T_2 to be time of flight in two cases, then the product of the two time of flights is directly proportional to?
a) R b) 1/R
c)$1/R^2$ d)R^2

20) Which of the following statements is FALSE for a particle moving in a circle with constant angular speed?

a) The acceleration vector points to the center of the circle
b) The acceleration vector is tangent to the circle
c) The velocity vector is tangent to the circle
d) The velocity and acceleration vectors are perpendicular to each other

21) A particle is projected at 60° to the horizontal with a kinetic energy K. The kinetic energy at highest point is?
a)K/2 b) K
c)zero d)K/4

22) Q45) A body of mass 0.5kg is projected under gravity with a speed of 98 m/s at angle of 60° with the horizontal. The change in momentum (in magnitude) of the body is?
a) 50.0 N-s b) 98.0N-s
c)24.5N-s d)49.0N-s

23) Two stones are projected with same velocity but makes different angles with the horizontal. Their ranges are equal. If the angle of projection of one is π/3 and its maximum height is h, then the maximum height of the other will be?
a) h/6 b) h/3
c)h/2 d)h/√2

24) A bullet is dropped from the same height when another bullet is fired horizontally. They will hit the ground?
a) depends upon mass of bullet
b) depends upon the observer
c)one after another
d)simultaneously

25) The maximum horizontal range of a projectile is 400m. The maximum height attended by it will be?
a) 100 m b) 200 m
c)400 m d)800 m

26) What determines the nature of the path followed by the particle?
a) Speed b) Velocity
c)Acceleration d)None of above

27) A particle moves in plane with constant acceleration in a direction different from the initial velocity. The path of the particle is a
a)straight line b) arc of a circle
c)parabola d)ellipse

28) An object is thrown along the direction inclined at an angle of 45° with horizontal direction. The horizontal range of the particle is?
a)vertical height
b) twice the vesicle height
c)thrice the vertical height
d)four times the vertical height

29) A gun fires two bullets at 60° and 30° with horizontal. The bullets strikes at same horizontal distance. The ratio of maximum height for the two bullets is in the ratio..
a) 2:1 b) 3:1
c)4:1 d)1:1

30) A hose pipe lying on the ground shoots a stream of water upward at an angle of 60° to the horizontal. The speed of the water is 20m/s as it leaves the hose. It will strike a wall 10m away at height of
a) 10.5 m b) 12.32 m
c) 10 m d) 20 m

31) A boy aims a gun at a bird from a point, at horizontal distance of 100 m. if gun can impart the velocity of 500 m/s to the bullet, what height above the bird must he aim his gun in order to hit it? (take $g=10\ m/sec^2$
a)100 cm b) 50 cm
c)40 cm d)20 cm

32) A particle is projected at 60° to the horizontal with a kinetic energy K. The kinetic energy at the highest point is?
a) K/2 b) K
c)zero d)K/4

33) Two balls are projected making an angle of 30° and 45° respectively with the horizontal. if both have same velocity at the highest points of their path, then the ratio of their velocities of projection is?
a) $\sqrt{3}:\sqrt{2}$ b) $\sqrt{2}:1$
c) $\sqrt{2}:\sqrt{3}$ d) $\sqrt{3}:2$

34) A stuntman plans to run across a roof top and then horizontally off it to land on the roof of the next building. The roof of the next building is 4.9 meter below the first one and 6.2 meter away from it. What should be his minimum roof top speed in m/s, so that he can successfully make the jump?
a) 3.1 b) 4.0
c)4.9 d)6.2

35) During projectile motion quantities that remain unchanged are ?
a) force and vertical velocity
b) acceleration and horizontal velocity

c) kinetic energy and acceleration
d) acceleration and momentum

36) A blind person after walking 10 steps in one direction, each of length 80cm, turns randomly to left or right by 90°. After waking of total of 40 steps, the maximum displacement of the person from its starting point can be
a) 32 m b) 16√2 m
c) 8√2 m d) 0 m

37) A particle is projected such that the horizontal range and vertical height are same. Then the angle of projection is
a) π/4 b) tan-1(4)
c) tan-1(1/4) d) π/3

38) A particle moves in a plane with a constant acceleration in a direction different from the initial velocity. The path of the particle is a
a) straight line b) arc of a circle
c) parabola d) ellipse

39) At the top of the trajectory of a projectile the directions of its velocity and acceleration are
a) parallel to each other
b) inclined at an angle of 45° to the horizontal
c) perpendicular to each other
d) none of the above statement is correct

40) The range of a particle when launched at an angle of 15° with horizontal is 1.5km. What is the range of the projectile when launched at an angle of 45° to the horizontal?
a) 1.5 km b) 3.0 km
c) 6.0 km d) 0.75 km

41) A Body moves 6 m north. 8 m east and 10m vertically upwards, what is its resultant displacement from initial position?
(a) $10\sqrt{2}m$ (b) $10m$
(c) $\frac{10}{\sqrt{2}}m$ (d) $10 \times 2\,m$

42) An Aero plane flies 400 m north and 300 m south and then flies 1200 m upwards then net displacement is
(a) 1200 m (b) 1300 m
(c) 1400 m (d) 1500 m

43) An athlete completes one round of a circular track of radius R in 40 sec. What will be his displacement at the end of 2 min. 20 sec
(a) Zero (b) 2R (c) 2πR (d) 7πR

44) A person travels along a straight road for half the distance with velocity v1 and the remaining half distance with velocity v2 The average velocity is given by
(a) v_1v_2 (b) $\frac{v_2^2}{v_1^2}$
(c) $\frac{v_1+v_2}{2}$ (d) $\frac{2v_1v_2}{v_1+v_2}$

45) The displacement-time graph for two particles A and B are straight lines inclined at angles of 30° and 60° with the time axis. The ratio of velocities of $V_A : V_B$ is
(a) 1:2 (b) 1:√3
(c) √3:1 (d) 1:3

46) A car travels from A to B at a speed of 20 km / hr and returns at a speed of 30 km / hr . The average speed of the car for the whole journey is?
(a) 25 km / hr (b) 24 km / hr (c) 50 km / hr (d) 5 km / hr

47) One car moving on a straight road covers one third of the distance with 20 km/hr and the rest with 60 km/hr. The average speed is?
(a) 40 km/hr (b) 80 km/hr
(c) $46\frac{2}{3}$ km/hr (d) 36 km/hr

48) A car moves for half of its time at 80 km/h and for rest half of time at 40 km/h. Total distance covered is 60 km. What is the average speed of the car
(a) 60 km / h (b) 80 km/h
(c) 120 km / h (d) 180 km/h

49) A train has a speed of 60 km/h. for the first one hour and 40 km/h for the next half hour. Its average speed in km/h is
(a) 50 (b) 53.33
(c) 48 (d) 70

50) Which of the following is a one dimensional motion
(a) Landing of an aircraft?
(b) Earth revolving around the sun
(c) Motion of wheels of a moving train
(d) Train running on a straight track

51) A particle is constrained to move on a straight line path. It returns to the starting point after 10 sec. The total distance covered by the particle during this time is 30 m. Which of the following statements about the motion of the particle is false
(a) Displacement of the particle is zero

(b) Average speed of the particle is 3 m/s
(c) Displacement of the particle is 30 m
(d) Both (a) and (b)

52) The ratio of the numerical values of the average velocity and average speed of a body is always
(a) Unity (b) Unity or less (c) Unity or more (e) Less than unity

53) If a car covers 2/5th of the total distance with v_1 speed and 3/5th distance with v_2 then average speed is?
(a) $\frac{1}{2}\sqrt{v_1 v_2}$ (b) $\frac{v_1+v_2}{2}$
(c) $\frac{2v_1 v_2}{v_1+v_2}$ (d) $\frac{5v_1 v_2}{3v_1+2v_2}$

54) The numerical ratio of displacement to the distance covered is always
(a) Less than one (b) Equal to one
(c) Equal to or less than one
(d) Equal to or greater than one

55) The correct statement from the following is
(a) A body having zero velocity will not necessarily have zero acceleration
(b) A body having zero velocity will necessarily have zero acceleration
(c) A body having uniform speed can have only uniform acceleration
(d) A body having non-uniform velocity will have zero acceleration

56) The instantaneous velocity of a body can be measured
(a) Graphically (b) Vectorally
(c) By speedometer (d) None of these

57) A body under the action of several forces will have zero acceleration
(a) When the body is very light
(b) When the body is very heavy
(c) When the body is a point body
(d) When the vector sum of all the forces acting on it is zero

58) The acceleration of a moving body can be found from
(a) Area under velocity-time graph
(b) Area under distance-time graph
(c) Slope of the velocity-time graph
(d) Slope of distance-time graph

59) The initial velocity of the particle is 10 m/sec and its retardation is $2m/s^2$. The distance moved by the particle in 5th second of its motion is
(a) 1 m (b) 19 m (c) 50 m (d) 75 m

60) A motor car moving with a uniform speed of 20 m / sec comes to stop on the application of brakes after travelling a distance of 10 m Its acceleration is
(a) 20 m / sec^2 (b) - 20m / sec^2
(c) - 40 m / sec^2 (d) + 2m / sec^2

61) The velocity of a body moving with a uniform acceleration of 2 m/sec^2 is 10 m / sec . Its velocity after an interval of 4 sec is
(a) 12 m / sec (b) 14 m / sec
(c) 16 m / sec (d) 18 m / sec

62) A particle starting from rest travels a distance x in first 2 seconds and a distance y in next two seconds, then
(a) y = x (b) y = 2x
(c) y = 3x (d) y = 4 x

63) The initial velocity of a body moving along a straight line is 7 m / s . It has a uniform acceleration of 4 m / s2 . The distance covered by the body in the 5th second of its motion is
(a) 25 m (b) 35 m
(c) 50 m (d) 85 m

64) Which of the following four statements is false
(a) A body can have zero velocity and still be accelerated
(b) A body can have a constant velocity and still have a varying speed
(c) A body can have a constant speed and still have a varying velocity
(d) The direction of the velocity of a body can change when its acceleration is constant

65) An alpha particle enters a hollow tube of 4 m length with an initial speed of 1 km/s. It is accelerated in the tube and comes out of it with a speed of 9 km/s. The time for which it remains inside the tube is
(a) 8 x10^{-3} s (b) 80 x10^{-3} s
(c) 800 x10^{-3} s (d) 8 x10^{-4} s

66) A car moving with a velocity of 10 m/s can be stopped by the application of a constant force F in a distance of 20 m. If the velocity of the car is 30 m/s, it can be stopped by this force in
(a) 20/3 m (b) 20 m (c) 60 m (d) 180 m

67) A car moving with a speed of 40 km/h can be stopped by applying brakes after atleast 2 m. If the same car is moving

with a speed of 80 km/h, what is the minimum stopping distance
(a) 8 m (b) 2 m (c) 4 m (d) 6 m

68) For a moving body at any instant of time
(a) If the body is not moving, the acceleration is necessarily zero
(b) If the body is slowing, the retardation is negative
(c) If the body is slowing, the distance is negative
(d) If displacement, velocity and acceleration at that instant are known, we can find the displacement at any given time in future

69) The engine of a car produces acceleration 4 m/s^2 in the car. If this car pulls another car of same mass, what will be the acceleration produced
(a) 8 m/s^2 (b) 2 m/s^2
(c) 4 m/s^2 (d) $\frac{1}{2} m/s^2$

70) A truck and a car are moving with equal velocity. On applying the brakes both will stop after certain distance, then
(a) Truck will cover less distance before rest
(b) Car will cover less distance before rest
(c) Both will cover equal distance
(d) None

71) The distance travelled by a particle is proportional to the squares of time, then the particle travels with
(a) Uniform acceleration
(b) Uniform velocity
(c) Increasing acceleration
(d) Decreasing velocity

72) Acceleration of a particle changes when
(a) Direction of velocity changes
(b) Magnitude of velocity changes
(c) Both of above
(d) Speed changes

73) Consider the acceleration, velocity and displacement of a tennis ball as it falls to the ground and bounces back. Directions of which of these changes in the process
(a) Velocity only
(b) Displacement and velocity
(c) Acceleration, velocity and displacement
(d) Displacement and acceleration

74) The path of a particle moving under the influence of a force fixed in magnitude and direction is
(a) Straight line (b) Circle
(c) Parabola (d) Ellipse

75) A body A moves with a uniform acceleration a and zero initial velocity. Another body B, starts from the same point moves in the same direction with a constant velocity v. The two bodies meet after a time t. The value of t is
(a) $\frac{2v}{a}$ (b) $\frac{v}{a}$
(c) $\frac{v}{2a}$ (d) $\sqrt{\frac{v}{2a}}$

76) An object accelerates from rest to a velocity 27.5 m/s in 10 sec then find distance covered by object in next 10 sec
(a) 550 m (b) 137.5 m
(c) 412.5 m (d) 275 m

77) The displacement of a particle is proportional to the cube of time elapsed. How does the acceleration of the particle depends on time obtained
(a) a ∝ t^2 (b) a ∝ 2t
(c) a ∝ t^3 (d) a ∝ t

78) What determines the nature of the path followed by the particle
(a) Speed (b) Velocity
(c) Acceleration (d) None of these

79) Two trains, each 50 m long are travelling in opposite direction with velocity 10 m/s and 15 m/s. The time of crossing is
(a) 2s (b) 4s
(c) $2\sqrt{3}s$ (d) $4\sqrt{3}s$

80) A 120 m long train is moving in a direction with speed 20 m/s. A train B moving with 30 m/s in the opposite direction and 130 m long crosses the first train in a time
(a) 6 s (b) 36 s
(c) 38 s (d) None of these

81) A 210 meter long train is moving due North at a of 25m/s. A small bird is flying due South a little above the train with speed 5m/s. The time taken by the bird to cross the train is
(a) 6s (b) 7s
(c) 9s (d) 10s

82) The distance between two particles is decreasing at the rate of 6 m/sec. If

these particles travel with same speeds and in the same direction, then the separation increase at the rate of 4 m/sec. The particles have speeds as
(a) 5 m/sec ; 1 m/sec
(b) 4 m/sec ; 1 m/sec
(c) 4 m/sec ; 2 m/sec
(d) 5 m/sec ; 2 m/sec

83) A boat moves with a speed of 5 km/h relative to water in a river flowing with a speed of 3 km/h and having a width of 1 km. The minimum time taken around a round trip is
(a) 5 min (b) 60 min
(c) 20 min (d) 30 min

84) Two bodies of different masses m_a and m_b are dropped from two different heights a and b. The ratio of the time taken by the two to cover these distances are
(a) a:b (b) b:a
(c) $\sqrt{a}:\sqrt{b}$ (d) $a^2:b^2$

85) A body falls freely from rest. It covers as much distance in the last second of its motion as covered in the first three seconds. The body has fallen for a time of
(a) 3 s (b) 5 s (c) 7 s (d) 9 s

86) An iron ball and a wooden ball of the same radius are released from the same height in vacuum. They take the same time to reach the ground. The reason for this is
(a) Acceleration due to gravity in vacuum is same irrespective of the size and mass of the body
(b) Acceleration due to gravity in vacuum depends upon the mass of the body
(c) There is no acceleration due to gravity in vacuum
(d) In vacuum there is a resistance offered to the motion of the body and this resistance depends upon the mass of the body

87) A ball P is dropped vertically and another ball Q is thrown horizontally with the same velocities from the same height and at the same time. If air resistance is neglected, then
(a) Ball P reaches the ground first
(b) Ball Q reaches the ground first
(c) Both reach the ground at the same time
(d) The respective masses of the two balls will decide the time

88) A stone dropped from the top of the tower touches the ground in 4sec. The height of the tower is about
(a) 80 m (b) 40 m
(c) 20 m (d) 160 m

89) A body falls from rest, its velocity at the end of first second is (g = 32ft / sec)
(a) 16 ft / sec (b) 32 ft / sec
(c) 64 ft / sec (d) 24 ft / sec

90) Two stones of different masses are dropped simultaneously from the top of a building
(a) Smaller stone hit the ground earlier
(b) Larger stone hit the ground earlier
(c) Both stones reach the ground simultaneously
(d) Which of the stones reach the ground earlier depends on the composition of the stone

91) A body starts to fall freely under gravity. The distances covered by it in first, second and third second are in ratio
(a) 1 : 3 : 5 (b) 1 : 2 : 3
(c) 1 : 4 : 9 (d) 1 : 5 : 6

92) P, Q and R are three balloons ascending with velocities U, 4U and 8U respectively. If stones of the same mass be dropped from each, when they are at the same height, then
(a) They reach the ground at the same time
(b) Stone from P reaches the ground first
(c) Stone from R reaches the ground first
(d) Stone from Q reaches the ground first

93) A body is thrown vertically upwards with a velocity u . Find the true statement from the following
(a) Both velocity and acceleration are zero at its highest point
(b) Velocity is maximum and acceleration is zero at the highest point
(c) Velocity is maximum and acceleration is g downwards at its highest point
(d) Velocity is zero at the highest point and maximum height reached is $u^2/2g$

94) A man throws a ball vertically upward and it rises through 20 m and returns to his hands. What was the initial velocity (u) of the ball and for how much time (T) it remained in the air
$[g = 10 m/s^2]$
(a) u = 10 m/s, T = 2s
(b) u = 10 m/s, T = 4s
(c) u = 20 m/s, T = 2s
(d) u = 20 m/s, T = 4s

95) Three different objects of masses m1, m 2 and m 3 are allowed to fall from rest and from the same point 'O' along three different frictionless paths. The speeds of the three objects, on reaching the ground, will be in the ratio of
(a) $m_1 : m_2 : m_3$ (b) $m_1 : 2m_2 : 3m_3$
(c) 1 : 1 : 1 (d) $\frac{1}{m_1} : \frac{1}{m_2} : \frac{1}{m_3}$

96) A body falls from a height h = 200m (at karak). The ratio of distance travelled in each 2 sec during t = 0 to t = 6 second of the journey is
(a) 1 : 4 : 9 (b) 1 : 2 : 4
(c) 1 : 3 : 5 (d) 1 : 2 : 3

97) The acceleration due to gravity on the planet A is 9 times the acceleration due to gravity on planet B. A man jumps to a height of 2m on the surface of A. What is the height of jump by the same person on the planet B
(a) 18m (b) 6m (c) 2/3 m (d) 2/9 m

98) A body, thrown upwards with some velocity reaches the maximum height of 50 m. Another body with double the mass thrown up with double the initial velocity will reach a maximum height of
(a) 100 m (b) 200 m
(c) 300 m (d) 400 m

99) From the top of a tower two stones, whose masses are in the ratio 1:2 are thrown one straight up with an initial speed u and the second straight down with the same speed u. Then, neglecting air resistance
(a) The heavier stone hits the ground with a higher speed
(b) The lighter stone hits the ground with a higher speed
(c) Both the stones will have the same speed when they hit the ground.
(d) The speed can't be determined with the given data.

100) When a ball is thrown up vertically with velocity V_0, it reaches a maximum height of 'h'. If one wishes to triple the maximum height then the ball should be thrown with velocity
(a) $\sqrt{3}V_0$ (b) $3V_0$ (c) $9V_0$ (d) $3/2\,V_0$

101) A man projects a coin upwards from the gate of a uniformly moving train. The path of coin for the man will be
(a) Parabolic (b) Inclined straight line
(c) Vertical straight line
(d) Horizontal straight line

102) An aero plane is flying horizontally with a velocity of 600 km/h at a height of 1960 m. When it is vertically at a point A on the ground, a bomb is released from it. The bomb strikes the ground at point B. The distance AB is
(a) 1200 m (b) 0.33 km
(c) 3.33 km (d) 33 km

103) A particle moves in a plane with constant acceleration in a direction different from the initial velocity. The path of the particle will be
(a) A straight line (b) An arc of a circle
(c) A parabola (d) An ellipse

104) A projectile fired with initial velocity u at some angle θ has a range R. If the initial velocity be doubled at the same angle of projection, then the range will be
(a) 2R (b) R / 2
(c) R (d) 4R

105) If the initial velocity of a projectile be doubled, keeping the angle of projection same, the maximum height reached by it will
(a) Remain the same (b) Be doubled
(c) Be quadrupled (d) Be halved

106) In the motion of a projectile freely under gravity, its
(a) Total energy is conserved
(b) Momentum is conserved
(c) Energy and momentum both are conserved
(d) None is conserved

107) The range of a projectile for a given initial velocity is maximum when the angle of projection is 45°. The range will be minimum, if the angle of projection is
(a) 90° (b) 180°
(c) 60° (d) 75°

108) A ball is thrown upwards and it returns to ground describing a parabolic path. Which of the following remains constant
(a) Kinetic energy of the ball
(b) Speed of the ball
(c) Horizontal component of velocity
(d) Vertical component of velocity

109) At the top of the trajectory of a projectile, the directions of its velocity and acceleration are
(a) Perpendicular to each other
(b) Parallel to each other
(c) Inclined to each other at an angle of 45°
(d) Antiparallel to each other

110) An object is thrown along a direction inclined at an angle of 45° with the horizontal direction. The horizontal range of the particle is equal to
(a) Vertical height
(b) Twice the vertical height
(c) Thrice the vertical height
(d) Four times the vertical height

111) A projectile thrown with a speed v at an angle θ has a range R on the surface of earth. For same v and θ, its range on the surface of moon will be
(a) R / 6 (b) 6R
(c) R / 36 (d) 36R

112) The greatest height to which a man can throw a stone is h. The greatest distance to which he can throw it, will be Motion in Two Dimension
(a) h/2 (b) h (c) 2h (d) 3h

113) A particle reaches its highest point when it has covered exactly one half of its horizontal range. The corresponding point on the displacement time graph is characterized by
(a) Negative slope and zero curvature
(b) Zero slope and negative curvature
(c) Zero slope and positive curvature
(d) Positive slope and zero curvature

114) At the top of the trajectory of a projectile, the acceleration is
(a) Maximum (b) Minimum (c) Zero (d) g

115) If a body A of mass M is thrown with velocity V at an angle of 30° to the horizontal and another body B of the same mass is thrown with the same speed at an angle of 60° to the horizontal. The ratio of horizontal range of A to B will be
(a) 1:3 (b) 1:1
(c) $1:\sqrt{3}$ (d) $\sqrt{3}:1$

116) Four bodies P, Q, R and S are projected with equal velocities having angles of projection 15°, 30°, 45° and 60° with the horizontal respectively. The body having shortest range is
(a) P (b) Q
(c) R (d) S

117) For a projectile, the ratio of maximum height reached to the square of flight time is ($g = 10\ ms^{-2}$)
(a) 5 : 4 (b) 5 : 2
(c) 5 : 1 (d) 10 : 1

118) If two bodies are projected at 30o and 60o respectively, with the same velocity, then
(a) Their ranges are same
(b) Their heights are same
(c) Their times of flight are same
(d) All of these

119) A ball is projected with velocity V_o at an angle of elevation 30°. Mark the correct statement
(a) Kinetic energy will be zero at the highest point of the trajectory
(b) Vertical component of momentum will be conserved
(c) Horizontal component of momentum will be conserved
(d) Gravitational potential energy will be minimum at the highest point of the trajectory

120) At what point of a projectile motion acceleration and velocity are perpendicular to each other
(a) At the point of projection
(b) At the point of drop
(c) At the topmost point
(d) Any where in between the point of projection and topmost point

121) The maximum horizontal range of a projectile is 400 m. The maximum value of height attained by it will be
(a) 100 m (b) 200
(c) 400 m (d) 800 m

122) The principle of conservation of linear momentum can be strictly applied during a collision between two particles provided the time of impact is
(a) Extremely small

(b) Moderately small
(c) Extremely large
(d) Depends on a particular case

123) A shell initially at rest explodes into two pieces of equal mass, then the two pieces' will
(a) Be at rest
(b) Move with different velocities in different directions
(c) Move with the same velocity in opposite directions
(d) Move with the same velocity in same direction

124) Two perfectly elastic particles P and Q of equal mass travelling along the line joining them with velocities 15 m/sec and 10 m/sec. After collision, their velocities respectively (in m/sec) will be
(a) 0, 25 (b) 5, 20
(c) 10, 15 (d) 20, 5

125) When two bodies collide elastically, then
(a) Kinetic energy of the system alone is conserved
(b) Only momentum is conserved
(c) Both energy and momentum are conserved
(d) Neither energy nor momentum is conserved

126) Two balls at same temperature collide. What is conserved
(a) Temperature (b) Velocity
(c) Kinetic energy (d) Momentum

ANSWERS WITH EXPLANATION

1) If there is no air resistance bomb will drop at a place exactly below the flying airplane, it is flying with constant velocity When we take air resistance in account horizontal velocity of bomb will reduce. So it will fall on surface behind the aero plane
Answer:(b)

2) Stone is thrown up against the gravitational acceleration we will take initial speed of stone 'u' negative gravitational acceleration positive
Let h be the height of bridge
$h = ut + ½(gt^2)$
$h = -4.9(2) + ½(9.8 \times 2^2)$
h=9.8 m
Answer :(c)

3) For maximum range, the angle of projection θ=45°

$$R = \frac{v_i^2 \sin 2\theta}{g} = \frac{20^2 \sin(2 \times 45)}{10} = 40m$$

Answer: (a)

4) Horizontal range is same when the angle of projection with horizontal is θ and (90° - θ)
Answer: (b)

5) In both the cases downward component of initial velocity is zero, thus both are free fall.
Both A and B will reach ground at same time
Answer:(b)

6) We know that range of two projectile having angle of projection θ and (90-θ) is same.
Answer:(b)

7) Let v be the initial velocity
Thus Initial kinetic energy $E = ½ m v^2$
At highest point Vertical velocity of ball will be zero.
horizontal velocity will be $v\cos\theta = \frac{v}{\sqrt{2}}$
Now kinetic energy at the highest point
$E' = ½ m \left(\frac{v}{\sqrt{2}}\right)^2 \rightarrow E' = ½ m v \times (1/2)$
$E' = E/2$
Answer: (b)

8) P=F. v=(6i-4j+3k) . (20i + 15j - 5k)
P=45 J/s
Answer: (a)

9) initial velocity of Bullet along vertical direction is zero
Time taken by bullet to cover horizontal distance t=Distance / speed of Bullet
t=100 / 1000=0.1sec
During this time, bullet will fall down due to gravitational acceleration
height 'h'=ut + ½ g t^2
$h = ½\, 10(0.1)^2$
h=0.05 m=5cm
Answer: (a)

10) (45° - θ) and (45° + θ) are complementary
We know that angle of projection of two projectile makes complementary angles, their ranges are equal. In this case, the

range will be same the ratio is 1:1
Answer:(d)

11) Formula for maximum range=V^2 / g
16000 (10)=V^2∴ v=400 m/s
Answer: (c)

12) Since displacement is along y-direction hence displacement vector S=10j
work done=F. S
Work done= (-2i + 15j + 6k)(10j)
Work Done=150J
Answer:(c)

13) $a = \frac{F}{m}$

$$a = \frac{6ti + 4tj}{3} \rightarrow a = 2ti + \left(\frac{4}{3}\right)tj$$

Velocity equation

$$v = u + at$$

$$v = 0 + \left[2ti + \left(\frac{4}{3}\right)tj\right]t$$

$$v = \left[2t2i + \left(\frac{4}{3}\right)t2j\right]$$

$$v = 2(3)2i + \left(\frac{4}{3}\right)(3)2j$$

$$v = 18i + 12j$$

Answer: (c)

14) The car moving with a constant velocity has no acceleration. Hence, it is an inertial frame
Answer: (b)

15) At uppermost point of projectile, velocity along vertical direction in zero, but have horizontal Velocity.
Acceleration is gravitational and vertically down
Thus angle is 90°
Answer: (b)

16) Equation for range is : $R = \frac{v_i^2 \sin 2\theta}{g} \rightarrow$

$$R = 2v_i^2 \sin\frac{2\theta\cos\theta}{g}$$

Equation for height $H = \frac{v_i^2 \sin^2\theta}{2g}$

By taking ratio of R/H we get

$$\frac{R}{H} = \frac{\frac{2u^2 \sin 2\theta\cos\theta}{g}}{\frac{u^2 \sin\theta}{2g}} \rightarrow \frac{R}{H} = \frac{4\cos\theta}{\sin\theta} \rightarrow \frac{R}{H}$$

$$= 4\cot\theta$$

Answer:(b)

17) according to formula for same range different angles of projection

$$t_1 t_2 = \frac{2R}{g} \rightarrow t_1 t_2 \propto R$$

Answer: (c)

18) let initial velocity be u at time t=0
vertical component of velocity $u\sin\theta = 0$ at max height
horizontal component will be $u\cos\theta = \frac{u}{2}$ which remains constant
Now $E \propto u^2$
Thus at max height E'=E/4
Answer: (c)

19) Equation for time of flight is $T = \frac{2u\sin\theta}{g}$
The angle of projection is different but range is same, then angle of projections are complimentary
Let one angle be θ then other is (90 - θ)

$$T_1 = \frac{2u\sin\theta}{g} \rightarrow T = \frac{2u\sin(90-\theta)}{g}$$

$$\rightarrow T_2 = \frac{2u\cos\theta}{g}$$

Taking product of T_1 and T_2 we get

$$T_1 T_2 = \frac{4u^2 \sin\theta\cos\theta}{g^2}$$

2sinθcosθ=sin2θ

$$T_1 T_2 = \frac{2u^2 \sin 2\theta}{g^2}$$

formula of range is

$$R = \frac{u^2 \sin 2\theta}{g}$$

from above
$T_1 T_2 = 2R/g$
Answer: (a)

20) (b)

21) During the flight horizontal component remain unchanged, and at maximum height vertical component becomes zero if u is velocity at t=0 then at maximum height velocity=ucons60=u/2

$$K \propto v^2$$

$$K'/K = \left(\frac{u}{2}\right)^2 / u_2$$

$$K' = K/4$$

Answer: (d)

22) Initial momentum
p_i=mu=0 (u=0)
When projected with velocity v=98 m/s at the time of projection momentum
p=mv=0.5×98=49.0 N-s
Answer:(d)

23) If range is same but angle of projection are different then $\theta_1 + \theta_2 = \pi/2$
If $\theta_1 = \pi/3$ then $\theta_2 = \pi/6$
formula for maximum height is

$$H = \frac{u^2 \sin^2\theta}{2g}$$

form formula for H

$$\frac{H}{H'} = \frac{\sin^2(\frac{\pi}{3})}{\sin^2(\frac{\pi}{6})} \rightarrow \frac{H}{H'} = \frac{(\frac{3}{4})}{\frac{1}{4}} = 3$$

$H' = H/3$

Answer: (b)

24) When bullet is fired horizontally there is no component of velocity along vertical direction therefore both the bullets will hit ground simultaneously
Answer:(d)

25) relation between range and maximum height is given by equation $\frac{H}{R} = \frac{1}{4}\tan\theta$
for maximum range θ=45

$$Thus\ R = \frac{H}{4} \Rightarrow R = \frac{400}{4} = 100\ m$$

Answer: (a)

26) When velocity changes, direction of path or nature of path changes,so, acceleration determines the nature of path
Answer:(c)

27) (c)

28) We know that $\frac{H}{R} = (\frac{1}{4})\ \tan\theta$
given $\theta = 45$

$$\therefore \frac{H}{R} = \frac{1}{4} \Rightarrow R = 4H$$

Answer: (d)

29) We know that $H = (\frac{R}{4})\ \tan\theta$
Thus $R = \frac{4H}{\tan\theta}$
Given Range is same

$$\frac{4H_1}{\tan 60} = \frac{4H_2}{\tan 30} \rightarrow \frac{H_1}{H_2} = \frac{\tan 60}{\tan 30} \rightarrow \frac{H_1}{H_2} = \frac{3}{1}$$

Answer:(b)

30) Horizontal displacement is 10 m
Horizontal component of water=20cos60=10 m/s
Time taken to travel 10 m, t=10/10=1 sec
Vertical component of velocity=20sin60=$10\sqrt{3}$
displacement along vertical h=ut - ½gt²

$$h = 10\sqrt{3}(1) - ½(10)(1) = 17.32 - 5 = 12.32\ m$$

Answer: (b)

31) Horizontal distance traveled by bullet in time t=d/v=100/500=1/5 sec
In time t=1/5 sec bullet will travel a vertically down ward distance of

$$h = ut + ½g(t)^2 \rightarrow h = ½g(t)^2$$

$$h = ½ \times 10 \times (\tfrac{1}{5})^2 = 0.2\ m = 20\ cm$$

Thus hunter should aim the gun 20cm above the target
Answer: (d)

32) Horizontal velocity v'=vcos60=v/2
Thus kinetic energy at top=K/4
Answer: (d)

33) let u be the velocity of first ball
horizontal component=ucos30
let v be the velocity of second ball
horizontal component=vcos45
given ucos30=vcos45
thus u/v=cos45/cos30

$$\frac{u}{v} = \frac{\frac{1}{\sqrt{2}}}{\frac{\sqrt{3}}{2}} \rightarrow \frac{u}{v} = \frac{1}{\sqrt{2}} \times \frac{2}{\sqrt{3}} \rightarrow \frac{u}{v} = \frac{\sqrt{2}}{\sqrt{3}}$$

Answer: (c)

34) Vertical velocity is zero thus time taken by the stuntman to travel a distance of 4.9 m is
4.9=½ 9.8 t²
t=1
Now stunt man has to travel distance of 6.2 m in one sec thus his speed must be 6.2m/s
Answer: (d)

35) (b)

36) Maximum displacement=distance

$$Distance = 40 \times 80 = 3200\ cm = 32m$$

Answer: (a)

37) $R = H \rightarrow \frac{2u^2 \sin\theta\cos\theta}{g} = \frac{u^2 \sin^2\theta}{2g} \rightarrow \tan\theta = 4 \rightarrow \theta = \tan^{-1}(4)$
Answer: (b)

38) (c)

39) Use formula for range $R = \frac{u^2 \sin 2\theta}{g}$
Answer: (b)

40) Displacement vector = 11i+11j
Answer:(b)

41) (a) $\vec{r} = x\hat{i} + y\hat{j} + +z\hat{k} \therefore \sqrt{x^2 + y^2 + z^2}$

$$r = \sqrt{6^2 + 8^2 + 10^2} = 10\sqrt{2}\ m$$

42) (a) An aero plane flies 400 m north and 300 m south so the net displacement is 100 m towards north
Then it flies 1200 m upward so r = $\sqrt{(100)^2 + (1200)^2} \rightarrow 1204m \approx 1200m$
The option should be 1204 m, because this value misleads one into thinking that

net displacement is in upward direction only

43) (b) Total time of motion is 2 min 20 sec = 140 sec. As time period of circular motion is 40 sec so in 140 sec. athlete will complete 3.5 revolution i.e., He will be at diametrically opposite point i.e., Displacement = 2R

44) (d) As the total distance is divided into two equal parts therefore distance averaged speed = $\frac{2v_1v_2}{v_1+v_2}$.

45) (c)

$$\frac{v_A}{v_B}=\frac{\tan\theta_A}{\tan\theta_B}=\frac{\tan 30^\circ}{\tan 60^\circ}=\frac{1/\sqrt{3}}{\sqrt{3}}=\frac{1}{3}$$

46) (b)

$$\text{distance average speed}=\frac{2v_1v_2}{v_1+v_2}$$

$$=\frac{2\times 20\times 30}{20+30}=\frac{120}{5}=24\ \text{km/hr}$$

47) (d)

$$\text{Average speed}=\frac{\text{total distance}}{\text{total time}}=\frac{x}{t_1+t_2}$$

$$\Rightarrow \frac{x}{\frac{\frac{x}{3}}{v_1}+\frac{\frac{2x}{3}}{v_2}}=\frac{1}{\frac{1}{3\times 20}+\frac{2}{3\times 60}}=36\frac{\text{km}}{\text{hr}}$$

48) (a)

$$\text{average speed}=\frac{v_1+v_2}{2}=\frac{80+40}{2}$$

$$=60\ \text{km/hr}$$

49) (b) Distance travelled by train in first 1 hour is 60 km and distance in next 1/2 hour is 20 km.

$$\text{so average speed}=\frac{\text{total distance}}{\text{total time}}$$

$$=\frac{60+20}{3/2}$$

$$=53.33\ \text{km/hr}$$

50) (d)

51) (c) Displacement of the particle will be zero because it comes back to its starting point

$$\text{average speed}=\frac{\text{total distance}}{\text{total time}}=\frac{30\text{m}}{10\text{sec}}$$

$$=3\ \text{m/s}$$

52) (b)

$$\frac{|\text{Average velocity}|}{|\text{average speed}|}=\frac{|\text{displacement}|}{|\text{distaance}|}=\leq 1$$

because displacement will either be equal or less than distance. It can never be greater than distance.

53) (d)

$$\text{average speed}=\frac{\text{total distance travelled}}{\text{total time taken}}$$

$$=\frac{x}{\frac{2x/5}{v_1}+\frac{3x/5}{v_2}}=\frac{5v_1v_2}{3v_1+2v_2}$$

54) (c) Since displacement is always less than or equal to distance, but never greater than distance. Hence numerical ratio of displacement to the distance covered is always equal to or less than one.

55) (a) When the body is projected vertically upward then at the highest point its velocity is zero but acceleration is not equal to zero (g = 9.8m / s2).

56) (a,c)

57) (a) $\vec{a}=\frac{\vec{F}}{m}$, If $\vec{F}=0$ then $\vec{a}=0$

58) (c) Acceleration a = tanθ , where θ is the angle of tangent drawn on the graph with the time axis.

59) (a)

$$S_n=u-\frac{a}{2}(2n-1)=10-\frac{2}{2}(2\times 5-1)$$

$$=1\ \text{meter}$$

60) (b)

$$\text{from } v^2=u^2+2as\rightarrow 0=u^2+2as$$

$$\Rightarrow a=-\frac{u^2}{2S}=\frac{-(20)^2}{2\times 10}=-20\ \text{m/s}^2$$

61) (d)

$$v=u+at\rightarrow 10+2\times 4=18\ \text{m/sec}$$

62) (c) If particle starts from rest and moves with constant acceleration then in successive equal interval of time the ratio of distance covered by it will be 1 : 3 : 5 : 7 ..... (2n -1)
i.e. ratio of x and y will be 1:3 i.e. $\frac{x}{y}=\frac{1}{3}\Rightarrow y=3x$

63) (a)

$$S_n=u+\frac{a}{2}[2n-1]\rightarrow$$

$$S_{5th}=7+\frac{4}{2}[2\times 5-1]=7+18=25\ \text{m}$$

64) (b) Constant velocity means constant speed as well as same direction throughout.

65) (d)

$$v^2=u+2as\Rightarrow (9000)^2-(1000)^2$$

$$=1\times a\times 4$$

$$\Rightarrow a = \frac{10^7 m}{s^2} \quad \text{Now } t = \frac{v-u}{a} = t$$
$$= \frac{9000-1000}{10^7}$$
$$= 8 \times 10^{-4} sec$$

66) (d)
$S \propto u^2$
→ If u becomes 3 times then S will become 9 times i. e. 9 × 20 = 180m

67) (a)
$$S \propto u^2 \rightarrow \therefore \frac{S_1}{S_2} = \left(\frac{u_1}{u_2}\right)^2 \Rightarrow \frac{2}{S_2} = \frac{1}{4} \Rightarrow S_2 8m$$

68) (d)

69) (b)
$F = m \times a \Rightarrow$ if force is constant then
$$a \propto \frac{1}{m}$$
So if mass is doubled acceleration becomes half.

70) If retarding force (F) and velocity (v) are equal then stopping distance ∝ m (mass of vehicle)
As $m_{car} < m_{truck}$ therefore car will cover less distance before coming to rest.

71) (a) $S \propto t^2$ (given) $\therefore s = Kt^2$
$$\text{Acceleration, } a = \frac{\Delta s^2}{\Delta t^2}$$
= 2k(constant), it means particle travls with uniform acceleration.

72) (c) Because acceleration is a vector quantities.

73) (b) Only direction of displacement and velocity gets changed, acceleration is always directed vertically downward.

74) (a)

75) (a)
$$\frac{1}{2}at^2 = vt \Rightarrow t = \frac{2v}{a}$$

76) (c)
$$u = 0, v = 27.5\frac{m}{s}, \text{ and } t = 10 \text{ sec}$$
$$\therefore a \frac{27.5-0}{10} = 2.75\ m/s^2$$
Now, the distance traveled in next 10 sec
$$S = ut + \frac{1}{2}at^2 = 27.5 \times 10 + \frac{1}{2} \times 2.75 \times 100$$
$$= 275 + 137.5 = 412.5\ m$$

77) (d) $x \propto t^3 \therefore x = kt^3$
$$\Rightarrow v = \frac{\Delta x}{\Delta t} = 3kt^2 \text{ and } a = \frac{\Delta v}{\Delta t} = 6kt$$
i. e., $a \propto t$

78) (d) The nature of the path is decided by the direction of velocity, and the direction of acceleration. The trajectory can be a straight line, circle or a parabola depending on these factors.

79) (b)
$$\text{Time} = \frac{\text{Total length}}{\text{Relative velocity}} = \frac{50+50}{10+15}$$
$$= \frac{100}{25} = 4 \text{ sec}$$

80) (d) Total distance Relative velocity
Hence
130 +120 = 250 m ⇒ 30 − (−20) = 50m/s
t = 250 / 50 = 5s

81) (b) Relative velocity of bird w.r.t train = 25+5=30m/s
time taken by the bird to cross train = t
$$= \frac{210}{30} = 7 \text{ sec}$$

82) (a) When two particles moves towards each other then
$$v_1 + v_2 = 6 \quad ...(i)$$
When these particles moves in the same direction then
$$v_1 - v_2 = 4 \quad ...(ii)$$
by solving $v_1 = 5$ and $v_2 = 1\ m/s$

83) (d) For the round trip he should cross perpendicular to the river
$$\therefore \text{Time for trip to that side} = \frac{1km}{4km/hr}$$
$$= 0.25\ hr$$
To come back, again he take 0.25 hr to cross the river. Total time is 30 min, he goes to the other bank and come back at the same point.

84) (c)
$$h = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$$
$$t_a = \sqrt{\frac{2a}{g}} \text{ and } t_b = \sqrt{\frac{2b}{g}} \Rightarrow \frac{t_a}{t_b} = \sqrt{\frac{a}{b}}$$

85) (b)
$$\frac{1}{2}g(3)^2 = \frac{g}{2}(2n-1) \Rightarrow n = 5s$$

86) (a)

87) (c) Vertical component of velocities of both the balls are same and equal to zero. so $t = \sqrt{\frac{2h}{g}}$

88) (a)

$h = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times (4)^2 = 80$ m

89) (b)
$v = g \times t = 32 \times t = 32$ ft/sec

90) (c)
$t = \sqrt{\frac{2h}{g}}$ and h and g are same

91) $S_n = u + \frac{g}{2}(2n-1)$ when $u = 0, S_1 : S_2 : S_3 = 1 : 3 : 5$

92) (b) It has lesser initial upward velocity.

93) (d)
At highest point $v = 0$, and $H_{max} = \frac{u^2}{2g}$

94) (d)
$u = \sqrt{2gh} \rightarrow \sqrt{2 \times 10 \times 20} = 20$ m/s
and $T = \frac{2u}{g} = \frac{2 \times 20}{10} = 4$ sec

95) (c) Speed of the object at reaching the ground $v = \sqrt{2gh}$
If heights are equal then velocity will also be equal

96) (c) $S_n \propto (2n-1)$
In equal time interval of 2 seconds, Ratio of distance = 1 : 3 : 5

97) (a)
$$H_{max} = \frac{u^2}{2g} \Rightarrow H_{max} \propto \frac{1}{g}$$
On planet B value of g is 1 / 9 times to that of A. So value of H_{max} will become 9 times i.e. 2×9 = 18 meter

98) (b)
$H_{max} \propto u^2$ It body projected with double velocity then maximum height will become four times i.e. 200 m

99) (c)

100) (a)
$$H_{max} \propto u^2 \therefore u \propto \sqrt{H_{max}}$$
i.e. to triple the maximum height, ball should be thrown with velocity $\sqrt{3}u$

101) (c) Because horizontal velocity is same for coin and the observer. So relative horizontal displacement will be zero.

102) (c) Horizontal displacement of the bomb AB = Horizontal velocity × time available
$$AB = u \times \sqrt{\frac{2h}{g}}$$
$$= 600 \times \frac{5}{18} \times \sqrt{\frac{2 \times 1960}{9.8}} = 3.33\ km$$

103) (c)

104) (d)
$$R = \frac{u^2 \sin 2\theta}{g} \rightarrow R \propto u^2$$
If initial velocity be doubled then range will become four times

105) (c)
$$H = \frac{u^2 \sin^2\theta}{2g} : H \propto u^2$$
initial velocity be doubled then maximum height reached by the projectile will quadrupled.

106) (a) An external force by gravity is present throughout the motion so momentum will not be conserved.

107) (a)
$$Range = \frac{u^2 \sin 2\theta}{g} : when\ \theta = 90°, R = 0$$
i.e. the body will fall at the point of projection after completing one dimensional motion under gravity.

108) (c) Because there is no accelerating or retarding force available in horizontal motion.

109) (a) Direction of velocity is always tangent to the path so at the top of trajectory, it is in horizontal direction and acceleration due to gravity is always in vertically downward direction. It means angle between $\vec{v}$ and $\vec{g}$ are perpendicular to each other.

110) (d)
$R = 4H \cot\theta$ if $\theta = 45°$ then $R = 4H\cot(45°) = 4H$

111) (b)
Range is given by $R = \frac{u^2 \sin 2\theta}{g}$
on moon $g_m = \frac{g}{6}$ Hence $R_m = 6R$

112) (c)
For greatest height, $\theta = 90°$
$$H_{max} = \frac{u^2 \sin^2(90)°}{2g} = \frac{u^2}{2g} = h(given)$$
$$R_{max} = \frac{u^2 \sin^2 2(45)°}{g} = \frac{u^2}{g} = 2h$$

113) (b)

114) (d) Acceleration through-out the projectile motion remains constant and equal to g.

115) (b) For complementary angles range will be equal.

116) (a) When the angle of projection is very far from 45° then range will be minimum.

117) (a)

$$H = \frac{u^2 \sin^2\theta}{2g} \text{ and } T = \frac{2u\sin\theta}{g}$$

$$\text{so } \frac{H}{T^2} = \frac{\frac{u^2\sin^2\theta}{2g}}{\left(\frac{2u\sin\theta}{g}\right)^2} \rightarrow \frac{g}{8} = \frac{5}{4}$$

118) (a) For complementary angles range is same

119) (c) Since horizontal component of velocity is constant, hence momentum is constant.

120) (c)

121) (b)

$$R_{max} = \frac{u^2}{g} = 400\text{m} \quad (\text{for } \theta = 45°)$$

$$H_{max} = \frac{u^2}{2g} = \frac{400}{2} = 200\text{m} \quad (\text{for } = 90°)$$

122) (a)

123) (c) According to law of conservation of linear momentum both pieces should possess equal momentum after explosion. As their masses are equal therefore they will possess equal speed in opposite direction.

124) (c) For a collision between two identical perfectly elastic particles of equal mass, velocities after collision get interchanged.

125) (c)

126) (d